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**Full Bridge LLC Resonant Converter Design**

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**Keywords:** DC-DC converters, LLC resonant converter , Gain of LLC resonant converter, Operation modes of LLC resonant converter, transformer design, efficiency evaluation.

# Abstract

The aim of this thesis is to investigate and design LLC resonant converter to provide a high voltage of 2000V for the operation of Radiofrequency Ion Thruster with high conversion efficiency and low losses. Radiofrequency Ion Thruster uses high frequency electromagnetic field for ionizing the xenon gas atoms which consists of free light electrons and heavy positive ions. An electric field is applied to accelerate the charged noble gas particles. It generates a thrust used to stabilize or move a satellite in orbit.

In this research work, LLC resonant converter has been designed and simulated with a high range of frequency and best conversion efficiency. During normal operation it works at resonance to provide maximum efficiency. When the output voltage varies, resonant tank gain is used to regulate it using above resonance or below resonance operation depending upon the variation in the voltage. Switching losses play a major role in the efficiency of converter. Soft switching technology is implemented to eliminate the turn on losses of switch. Also the soft switching is not load dependent and even at light load the LLC can maintain the soft switching that gives high efficiency at light load regions.

Voltage regulation has been achieved by controlling the gain of resonant tank with the help of switching frequency. Transformer provides the main part of the gain and isolated DC to DC conversion as well.

There is not proper way to choose the resonant tank components' size. Therefore, a MATLAB GUI interface has been designed to choose the best operating point and components' size, considering the required output voltage and power for higher efficiency. Which also provides the required frequency modulation range for regulating the output.

Transformer design is also implemented considering high efficiency and minimum skin effect losses. Core design is also very critical design step avoiding the saturation of core at high frequency when a variable frequency control is implemented.

# Sommario

Lo scopo di questa tesi è di indagare e progettare un convertitore risonante LLC per fornire un'alta tensione di 2000 V per il funzionamento di propulsori ionici a radiofrequenza con alta efficienza di conversione e basse perdite. Il propulsore ionico a radiofrequenza utilizza un campo elettromagnetico ad alta frequenza per ionizzare gli atomi di gas xeno che sono costituiti da elettroni di luce libera e ioni positivi pesanti. Un campo elettrico applicato per accelerare le particelle di gas nobile cariche. Genera una spinta utilizzata per stabilizzare o spostare un satellite in orbita.

In questo lavoro di ricerca, il convertitore risonante LCC è stato progettato e simulato con un'elevata gamma di frequenza e la migliore efficienza di conversione. Durante il normale funzionamento lavora in risonanza per fornire la massima efficienza. Quando la tensione di uscita varia, il guadagno del serbatoio risonante viene utilizzato per regolarlo utilizzando il funzionamento al di sopra o al di sotto della risonanza a seconda della variazione della tensione. Le perdite di commutazione svolgono un ruolo importante nell'efficienza del convertitore. La tecnologia di commutazione morbida è implementata per eliminare le perdite di dell'interruttore. Inoltre, la commutazione graduale non dipende dal carico e anche a carico leggero, LLC può mantenere la commutazione graduale che offre un'elevata efficienza nelle regioni di carico leggero.

La regolazione della tensione è stata ottenuta controllando il guadagno del serbatoio di risonanza con l'aiuto della frequenza di commutazione. Il trasformatore fornisce la parte principale del guadagno e la conversione isolata da CC a CC.

Non esiste un modo corretto per scegliere la dimensione dei componenti del serbatoio risonante. Pertanto, è stata progettata un'interfaccia GUI MATLAB per scegliere il punto operativo e le dimensioni dei componenti migliori, tenendo conto della tensione e della potenza di uscita richieste per una maggiore efficienza. Che fornisce anche la gamma di modulazione di frequenza richiesta per la regolazione dell'uscita.

Il design del trasformatore è implementato anche considerando l'elevata efficienza e le perdite minime dovute all'effetto pelle. La progettazione di nucleo è anche una fase di progettazione molto critica. È progettato per evitare la saturazione della saturazione del nucleo ad alta frequenza quando viene implementato il controllo della frequenza variabile.

## Extended Abstract

In LLC resonant converters, a switching bridge generates a voltage pulse and that voltage pulse excites resonant tank and creating sinusoidal current. A resonant or sinusoidal current get transferred and scaled to a secondary side rectifier and then filtered by the output capacitors and that's a basic conversion from DC to AC in a resonant fashion. In LLC resonant converter the current always starts with the reverse polarity from the primary switches or MOSFETs and that's what creates the zero voltage switching.

Similarly, on secondary side rectifier, each diode has a half wave sinusoidal waveform and it starts and ends at zero which provides soft switching.

The gain from input to output is basically the gain of the switching bridge and then multiplied by the gain of resonant tank and multiplied by transformer ratio. Now the bridge gain and the transformer turns ratio are fixed, components the variable gain is only resonant tank voltage gain and that's a function of three parameters ( $Q$ ,  $m$ ,  $FX$ ).

$Q$  is quality factor and that's changes as a function of the load so the high load means a high  $Q$  value so as the output current changes, resonant tank gain also would change.

$m$  is a design parameter that relating the total primary inductance to the resonance inductance. Value of  $m$  does not change with output.  $FX$  is the normalized switching frequency and that's equal to the switching frequency divided by the resonant frequency.  $FX$  or the switching frequency is the control parameter in order to achieve the voltage regulation required.

MATLAB is used to plot different gains with the switching frequency for different curves. This is the somehow complex procedure to relate frequency to the gain by just a simple hand calculation. Further each curve represents a load value or in other words  $Q$  value. Each load has its own peak value and to the right of the peak there is an inductive operation and to the left of that peak is capacitive operation.

# Contents

Acknowledgement.....	i
Abstract .....	ii
Sommario .....	iii
Extended Abstract.....	iv
List of Figures .....	viii
List of Table .....	ix
1 Introduction.....	1
1.1 DC to DC converter.....	1
1.1.1 Non Isolated Converters .....	1
1.1.2 Isolated Converters .....	1
1.1.3 Overview of LLC Resonant Converter .....	2
1.1.4 Converter Bridge .....	3
1.1.5 Resonant Tank.....	3
1.1.6 Transformer .....	3
1.1.7 Rectifier Circuit.....	4
1.1.8 Load Filter .....	4
2 Gain of LLC Resonant Converter.....	5
2.1 Transformer Gain .....	5
2.2 Simplified Model.....	6
2.3 Reflected Load Resistance .....	6
2.4 Resonant Tank Gain .....	8
2.4.1 Quality Factor 'Q' .....	8
2.4.2 'm' Parameter .....	8
2.4.3 Normalized Switching Frequency ' $Fx$ ' .....	8
2.4.4 Tank Gain as a Function of Control Parameters (Q, $Fx$ and m). .....	9
3 Modes of Operation .....	11
3.1 At Resonance .....	11
3.1.1 Analytical Expression for the Magnetising Current at Resonance: .....	12
3.1.2 Analytical Expression for Transformer Primary Current.....	13
3.2 Below Resonance .....	14
3.2.1 Analytical Expression for the Magnetising Current Below Resonance:.....	15

3.2.2	Analytical Expression for Transformer Primary Current Below Resonance:.....	17
3.3	Above Resonance.....	18
3.3.1	Analytical expression for transformer primary current above resonance: .....	20
3.4	Operation Principal .....	20
3.4.1	Power Delivery Period 1 ( $t_0 \rightarrow t_4$ ) .....	21
3.4.2	Soft Switching Period for Q1 ( $t_4 \rightarrow t_6$ ).....	22
3.4.3	Power Delivery Period 2 ( $t_6 \rightarrow t_{10}$ ).....	22
3.4.4	Soft Switching Period for Q1 ( $t_{10} \rightarrow t_{12}$ ) .....	23
4	Design Steps.....	24
4.1	Selecting The $Q_{max}$ Value .....	25
4.2	Selecting The m Value .....	26
4.3	Finding The Minimum Normalized Switching Frequency.....	27
4.4	Required Voltage Gain vs Available Voltage Gain.....	28
4.5	Design Flow Chat.....	29
4.6	Power Delivery Cycle.....	30
4.7	Freewheeling Cycle .....	30
5	Transformer Design.....	31
5.1	The Ideal Transformer.....	32
5.1.1	The Magnetizing Inductance.....	33
5.1.2	Leakage Inductances .....	35
5.2	Turns Per Volt Calculation .....	37
5.3	Primary Conductor Design.....	39
5.4	Secondary Conductor Design.....	39
5.5	Core Design.....	39
6	Bridge and Rectifier Selection.....	40
6.1	Bridge Selection .....	40
6.1.1	Half Bridge.....	40
6.1.2	Full Bridge .....	41
6.1.3	Selection of Full Bridge vs Half Bridge Topology for LLC Converter .....	42
6.2	Rectifier Selection .....	42
6.2.1	Halaf Wave Rectifier .....	42
6.2.2	Full Wave Rectifier.....	43
6.3	Rectifier Selection For our Design of LLC Converter .....	44

7	Efficiency .....	45
7.1	Efficiency At Resonance .....	45
7.2	Efficiency Below Resonance .....	45
7.3	Efficiency Above Resonance .....	45
8	Conclusion.....	46
9	Bibliography.....	49



# List of Figures

- Figure 1.1 Isolated DC to DC converter ..... 1
- Figure 1.2 Full bridge LLC converter ..... 3
- Figure 2.1 Simplified model of LLC converter ..... 6
- Figure 3.1 At resonance ..... 11
- Figure 3.2 Below resonance ..... 14
- Figure 3.3 Above resonance ..... 18
- Figure 3.4 Half Bridge LLC Converter ..... 21
- Figure 3.5 Operating Cycles ..... 21
- Figure 4.1 Tank gain with varying Q value and fixed m value ..... 25
- Figure 4.2 Tank gain at low value of m ..... 26
- Figure 4.3 Tank gain at high value of m ..... 26
- Figure 4.4 Tank gain at moderate value of m ..... 27
- Figure 4.5 Design flow chart ..... 29
- Figure 4.6 Power delivery cycle ..... 30
- Figure 4.7 Freewheeling cycle ..... 30
- Figure 5.1 A two-winding transformer [4] ..... 31
- Figure 5.2 Magnetic circuit that models the two-winding transformer of Figure 5.1 [4] ..... 31
- Figure 5.3 Ideal transformer symbol [4] ..... 32
- Figure 5.4 Transformer model including magnetizing inductance [4] ..... 34
- Figure 5.5 Leakage flux: (a) transformer geometry, (b) an equivalent system [2] ..... 36
- Figure 5.6 Two-winding transformer equivalent circuit [4] ..... 37
- Figure 6.1 Half bridge topology ..... 40
- Figure 6.2 Full bridge topology ..... 41
- Figure 6.3 Halaf Wave Rectifier ..... 42
- Figure 6.4 Full Wave Rectifier ..... 43
- Figure 7.1 Efficiency at resonance (blue) Vs efficiency below resonance (orange) ..... 45

# List of Table

Table 4-1 Design Specifications ..... 24  
Table 4-2 Resonant Tank Parameter ..... 28

# 1 Introduction

## 1.1 DC to DC converter

DC to DC converters are used to convert one DC voltage level to another DC voltage level by storing the input energy and then releasing it to a different voltage level. Depending upon the voltage level to convert it can be a boost operation or buck operation. These converters are used in wide range of applications. However, Efficiency, voltage ripple and load transient response are very important factors to be considered while designing a DC to DC converter. DC to DC converters can be isolated and non-isolated converter. In this chapter LLC resonant converter is briefly discussed.

### 1.1.1 Non Isolated Converters

In non-isolated DC-DC converter, the input and output terminals are connected to a common ground. These type of converters are used when voltage is not so high. For higher voltages isolation is necessary for protection. Four main types of non-isolated converter topology are buck, boost, buck-boost and Cuk converter.

### 1.1.2 Isolated Converters

In isolated DC-DC converter, the input and output terminals are isolated and hence provide good protection and also avoid noises. When voltage is electrically high we will use isolated converter. The demand for isolated DC-DC converters is growing, including telecommunications, data centers, battery chargers, industries, and aerospace applications. Isolated DC-DC converters with galvanic isolation are also necessary in some cases. Even if a human being touches one terminal of the power supply with galvanic isolation, no leakage current will flow through the human body to the ground. Therefore, for safety consideration, galvanic isolation is a fundamental requirement in some applications [1].

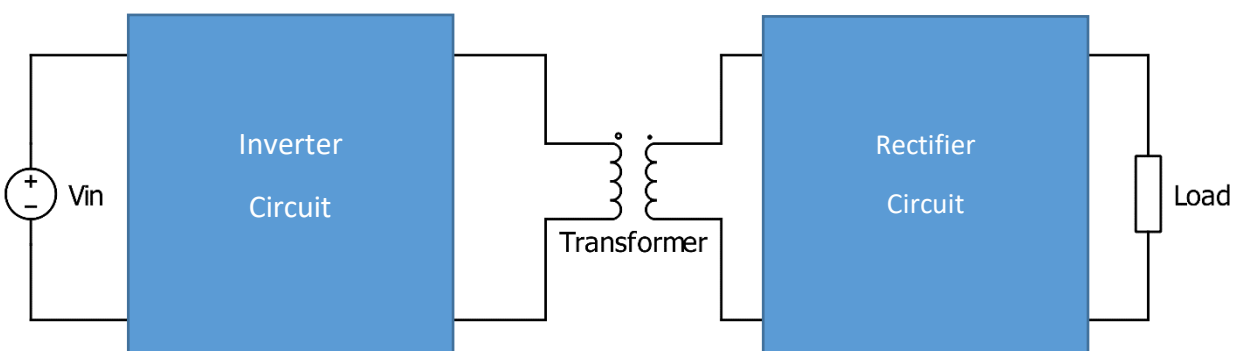


Figure 1.1 Isolated DC to DC converter

The mainly used are half-bridge, full-Bridge, fly-back, forward, and push-pull and LLC resonant converters.

1. **Flyback converter:** These converters are used for less than 100W of power. It is used for AC to DC conversion and DC to AC conversion. It is a buck boost converter with the inductor split to form a transformer, so that voltage ratios are multiplied with an additional advantage of isolation.
2. **Forward converter:** These converters are slightly modified versions of the Flyback converter. The forward converter is a famous circuit for low and medium power levels, up to about 500 W. It has one transistor, as does the flyback, but it requires a smaller transformer core.
3. **Push-pull converter:** This converter uses a transformer with a center tap. The push-pull converter is used for medium to high power requirements, typically up to 1000W. Advantages include transistor drive circuits with a common point and a relatively small transformer core because it is excited in both directions.
4. **Half Bridge converter:** Half-bridge converter consists of DC into AC conversion stage, and this AC is pumped into the high-frequency transformer, and then the rectification takes place. The limitation is that it is applied for small ratings as for a particular input voltage, the output will be half of the input voltage, and switch stress also becomes very large here. So, it is used for a power range up to 500W.
5. **Full Bridge converter:** Extension of the half-bridge converter. The full-bridge converter is often the circuit of choice for high-power applications, up to about 2000 W. The voltage stress on the transistors is limited to input voltage.
6. **Dual Active Bridge converter:** The converter employs two full bridges on each side of the isolation transformer. By controlling the phase shift angle between the primary and secondary, bi-directional power flow can be achieved. Zero voltage switching is an attractive feature of a DAB converter.
7. **Resonant Converters:** With the help of a resonant circuit, soft switching can be achieved by operating at the resonant frequency. High efficiency can be achieved due to very minimal losses when compared to DAB.

### 1.1.3 Overview of LLC Resonant Converter

LLC resonant converter is a DC to DC converter which provides high conversion efficiency and low EMI. However, it is complex to make an optimal design with variable frequency. Figure 1.1 shows a full bridge LLC resonant converter with full bridge rectifier.

The Input DC voltage is applied to switching circuit which generates the pulsating DC voltage using the soft switching. Then Resonant tank filters the harmonics and convert the pulsating DC into sinusoidal to feed the transformer. Resonant tank feeds this sinusoidal voltage to transformer which provides its scaling and primary and secondary isolation as well. Then rectifier circuit rectifies the input and provides the required output DC voltage.

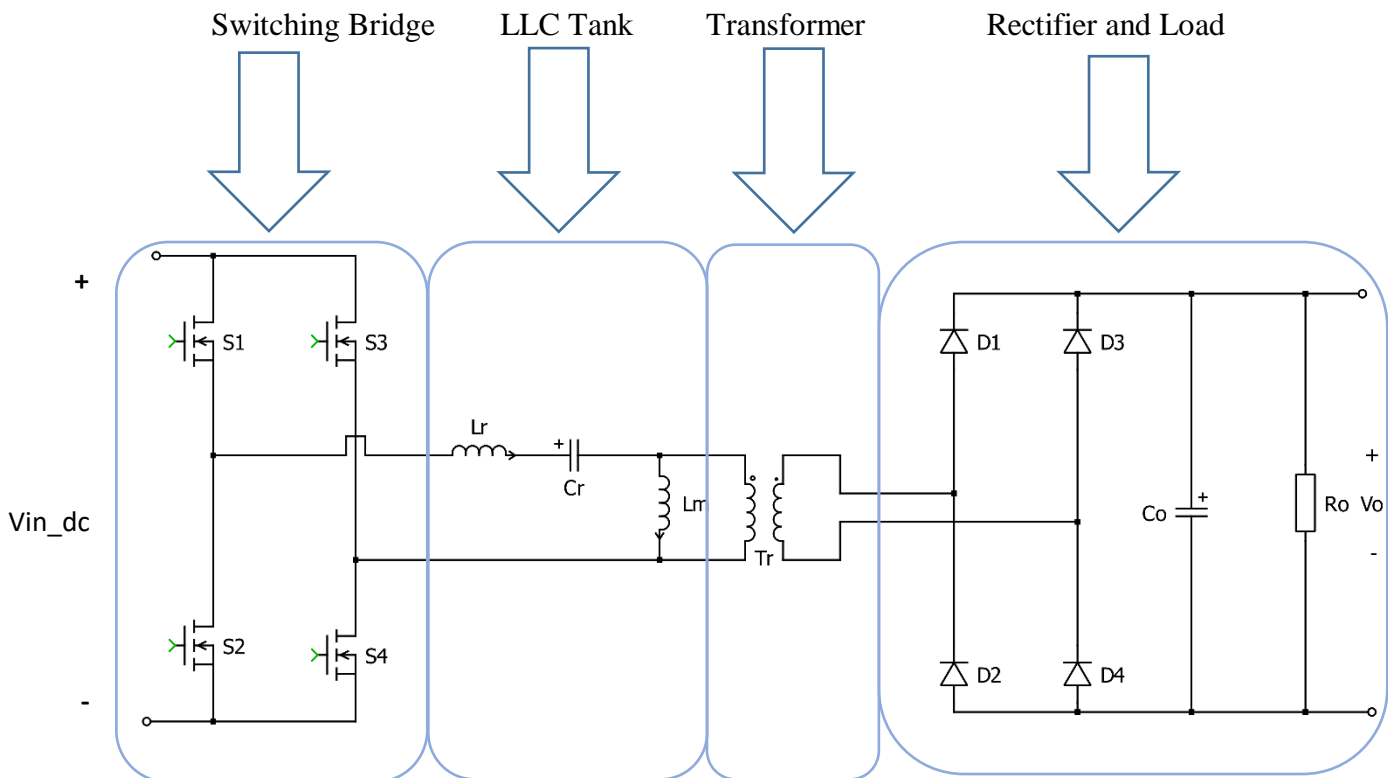


Figure 1.2 Full bridge LLC converter

### 1.1.4 Converter Bridge

The switching circuit can be half bridge (HB) or full bridge (FB). Both are having some advantages and disadvantages. In case of current, FB has advantage of lower current since full voltage swing is applied. In LLC converter, in case of HB for quite narrow voltage regulation and quite wide frequency range is needed.

### 1.1.5 Resonant Tank

The resonant tank consists of two inductors and a capacitor. which provide the sinusoidal current wave to the transformer. The parallel inductor provides the soft switching and reduces the losses. However, their values are chosen wisely to operate the converter at the best operating range considering the best efficiencies at the changing load and input voltage condition.

### 1.1.6 Transformer

A high frequency transformer is used for LLC converter. It is very critical step of LLC converter to design a suitable transformer considering the core and copper losses of transformer at high

frequency without saturating the core of transformer. Area product method or power handling method can be used to design the core of transformer. In this paper we have used power handling method to design the core of transformer. Skin depth is considered to choose the size of the conductor to minimize the copper losses of the conductor.

### 1.1.7 Rectifier Circuit

Rectifier circuit is used to convert the AC signal to the DC signal. Half wave rectifier (HWR) and full wave rectifier is used for this purpose. In this paper we are using full wave rectifier which is having the high efficiency and low ripple factor. HWR is not used since high ripple factor is produced and power is delivered only during half of the cycle. Its advantage is only its cheap ,simple and easy to construct.

### 1.1.8 Load Filter

At the load a capacitor filter is used to provide the smooth DC voltage at the output. Its value is chosen to provide the load ripple voltage less than 0.2% of the output voltage by using [4].

$$C = \frac{\Delta I}{2f * V_{ripple}} \quad (1)$$

Where C is the output filter capacitance value.

*V<sub>ripple</sub> is the acceptable ripple voltag*

## 2 Gain of LLC Resonant Converter

Voltage gain of LLC converter is the product of gains of three stages :

switch gain\* Resonant tank gain \* Transformer gain.

Switch gain and transformer gain are fixed and these don't change with the operation. The only control parameter is the resonant tank gain. The main part of the converter gain is provided by the transformer. By setting its turn ratio, we can achieve desired gain of the converter. The disadvantage is the losses in the transformer.

### 2.1 Transformer Gain

The main part of the converter gain is provided by the transformer. By setting its turn ratio we can achieve desired gain of the converter. It is fixed gain and it doesn't change during the operation. The disadvantage is the losses in the transformer which can be minimized by the resonant circuit by generating the sinusoidal pulse.

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} \quad (2)$$

Where

$V_s$  is the transformer Secondary voltage

$V_p$  is the transformer Primary voltage

$N_s$  is the secondary winding turns

$N_p$  is the primary winding turns

Two main losses in the transformer are following:

- i. Copper losses

$$I^2 * R \text{ Losses}$$

where ' $I$ ' is the current flowing through the transformer winding and ' $R$ ' is the resistance of the winding.

- ii. Core losses

Core losses are produced as a result of magnetizing and demagnetizing of the core of transformer during normal operation. They can be minimized by properly choosing the suitable ferromagnetic material .

## 2.2 Simplified Model

To analyze the behavior resonant tank gain, a simplified model is used. The transformer secondary resistance is calculated and reflected to the primary side. The whole circuit is replaced by an equivalent resistance. As shown in the figure 2.1.

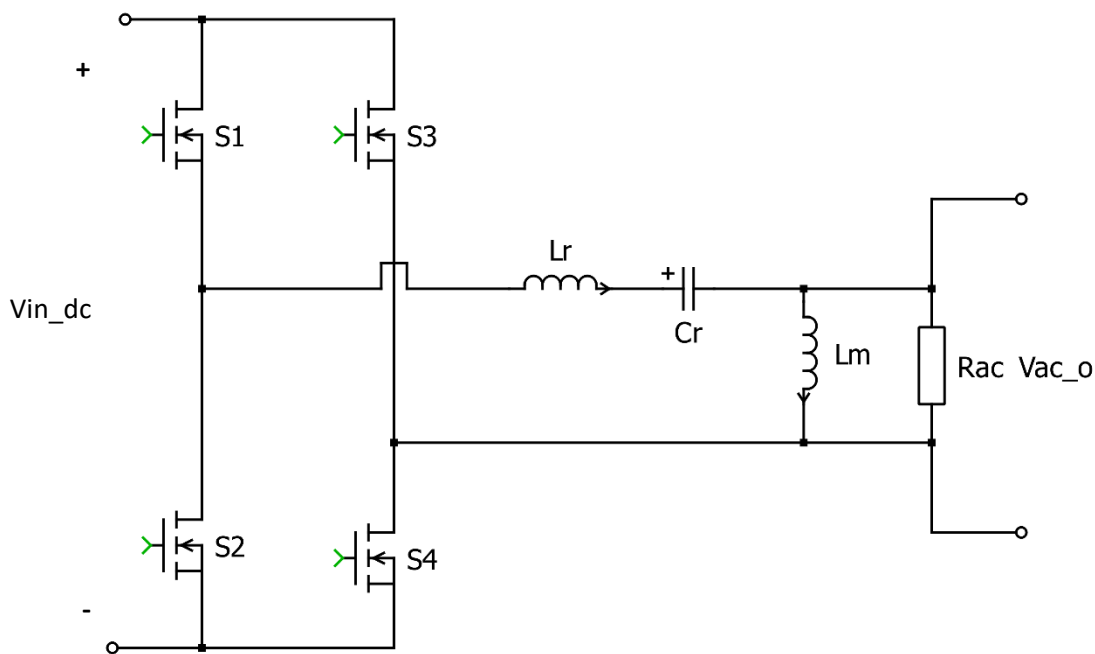


Figure 2.1 Simplified model of LLC converter

## 2.3 Reflected Load Resistance

To determine the simplified model of LLC converter the equivalent resistance seen from the primary side of transformer is calculated first, which is called reflected load resistance.

The output current ( $I_{dc}$ ) of the rectifier is given by [4].

$$I_{dc} = \frac{1}{\pi} I_p \int_0^{\pi} \sin \omega t dt \quad (3)$$

Where  $I_p$  is the peak value of the rectifier output current.



$$I_{dc} = \frac{2}{\pi} I_p \quad (4)$$

The secondary current of transformer ( $I_{ac}$ ) is fed to the rectifier which is given by

$$I_{ac} = I_p \sin \omega t \quad (5)$$

From (4) & (5)

$$I_{ac} = \frac{\pi}{2} \cdot I_{dc} \cdot \sin \omega t \quad (6)$$

The Secondary voltage ( $V_{ac}$ ) of the transformer is given by

$$V_{ac} = \begin{cases} V_0 & \text{if } \text{positive cycle} \\ -V_0 & \text{if } \text{negative cycle} \end{cases}$$

We will take the fundamental harmonic of  $V_{ac}$  which is dominant,

$$V_{ac} = \frac{4V_0}{\pi} \sin \omega t$$

Therefore secondary side equivalent resistance ( $R'_{ac}$ ) of transformer is given by

$$R'_{ac} = \frac{V_{ac}}{I_{ac}} = \frac{V_{ac}}{I_{ac}}$$

$$R'_{ac} = \frac{8}{\pi^2} \cdot R_0 \quad (7)$$

Shifting to the primary side of transformer

$$R_{ac} = n^2 \cdot R'_{ac} = n^2 \cdot \frac{8}{\pi^2} R_0$$

$$\therefore \left[ \text{where } n = \frac{N_p}{N_s} \right]$$

$$R_{ac} = \frac{8}{\pi^2} \cdot n^2 R_0 \quad (8)$$

## 2.4 Resonant Tank Gain

It is the function of three parameters Q,  $F_x$  and m parameter.

### 2.4.1 Quality Factor 'Q'

Q is the quality factor and depends on load. High load means a high Q value, as output current changes the resonant tank gain also changes [2].

$$Q = \frac{\sqrt{\frac{L_r}{C_r}}}{R_{ac}} \quad (9)$$

### 2.4.2 'm' Parameter

m is the function of primary inductance and it's the design parameter

$$m = \frac{L_m + L_r}{L_r} \quad (10)$$

It doesn't change with the operation.

### 2.4.3 Normalized Switching Frequency ' $F_x$ '

$F_x$  is the normalized frequency

$$F_x = \frac{f_s}{f_r} \quad \text{where } f_r = \frac{1}{2\pi\sqrt{L_r C_r}} \quad (11)$$

Its control parameter in order to achieve the voltage regulation required.

#### 2.4.4 Tank Gain as a Function of Control Parameters ( $Q$ , $F_x$ and $m$ ).

The voltage gain of resonant tank is calculated as following from the simplified model (figure 3.1).

$$\frac{V_{o\_ac}(s)}{V_{in\_ac}(s)} = \frac{R_{ac}/L_m s}{L_r s + \frac{1}{s C_r} + R_{ac}/L_m s} \quad (12)$$

Solving and by replacing  $s$  by  $j\omega$

$$\frac{V_{o\_ac}(s)}{V_{in\_ac}(s)} = \frac{L_m C_r \omega^2}{(L_r C_r \omega^2 + L_m C_r \omega^2 - 1) + j \left( \frac{w L_m \cdot w L_r \cdot C_r / w}{R_{ac}} - \frac{L_m \omega}{R_{ac}} \right)} \quad (13)$$

Now,

To convert the tank gain into control parameters we have

Equivalent control parameter from the numerator for equation (13)

$$L_m C_r \omega^2 = L_m C_r \omega^2 + (L_r C_r \omega^2 - L_r C_r \omega^2)$$

$$L_m C_r \omega^2 = \omega^2 L_r C_r \left[ \frac{L_m + L_r}{L_r} - 1 \right]$$

$$L_m C_r \omega^2 = \frac{\omega^2}{\frac{1}{L_r C_r}} \left[ \frac{L_m + L_r}{L_r} - 1 \right]$$

$$L_m C_r \omega^2 = \left[ \frac{\omega/2\pi}{\frac{1}{2\pi\sqrt{L_r C_r}}} \right]^2 \left[ \frac{L_m + L_r}{L_r} - 1 \right]$$

$$L_m C_r \omega^2 = \left( \frac{f_s}{f_r} \right)^2 [m - 1] \quad (14)$$

$$F_x = \frac{f_s}{f_r}$$

$$m = \frac{L_m + L_r}{L_r}$$

Now, equivalent control parameter in real part of denominator in equation (13)

$$L_r C_r \omega^2 + L_m C_r \omega^2 - 1 = \omega^2 L_r C_r \left[ \frac{L_m}{L_r} + 1 \right] - 1$$

$$L_r C_r \omega^2 + L_m C_r \omega^2 - 1 = F_x^2 (m) - 1 \quad (15)$$

Now, equivalent control parameter imaginary part of denominator in equation (13)

$$\frac{L_m L_r C_r \omega}{R_{ac}} - \frac{L_m \omega}{R_{ac}} = \frac{L_m \omega}{R_{ac}} (L_r C_r \omega^2 - 1)$$

$$\frac{L_m L_r C_r \omega}{R_{ac}} - \frac{L_m \omega}{R_{ac}} = \omega \sqrt{L_r C_r} \times \frac{\sqrt{L_r}}{R_{ac}} \times \left( \frac{L_m + L_r}{L_r} - 1 \right) (L_r C_r \omega^2 - 1)$$

$$\frac{L_m L_r C_r \omega}{R_{ac}} - \frac{L_m \omega}{R_{ac}} = F_x \cdot Q \cdot (m - 1) (F_x^2 - 1) \quad (16)$$

From (14), (15) and (16)

$$\frac{V_{0\_ac}(s)}{V_{in\_ac}(s)} = \frac{F_x^2 (m - 1)}{\sqrt{(m \cdot F_x^2 - 1)^2 + F_x^2 (F_x^2 - 1)^2 \cdot (m - 1)^2 Q^2}}$$

## 3 Modes of Operation

There are three modes of operation of LLC converter

1. At resonance
2. Below resonance
3. Above resonance

### 3.1 At Resonance

1. Complete power delivery operation
2. ZVS is achieved \*(losses saved by hard switching should be calculated)
3. CCM on Secondary side. (continues conduction mode)
4. Rectifier are soft switched
5. Optimum efficiency. Therefore, transformer turn ratio is designed to operate at this point.

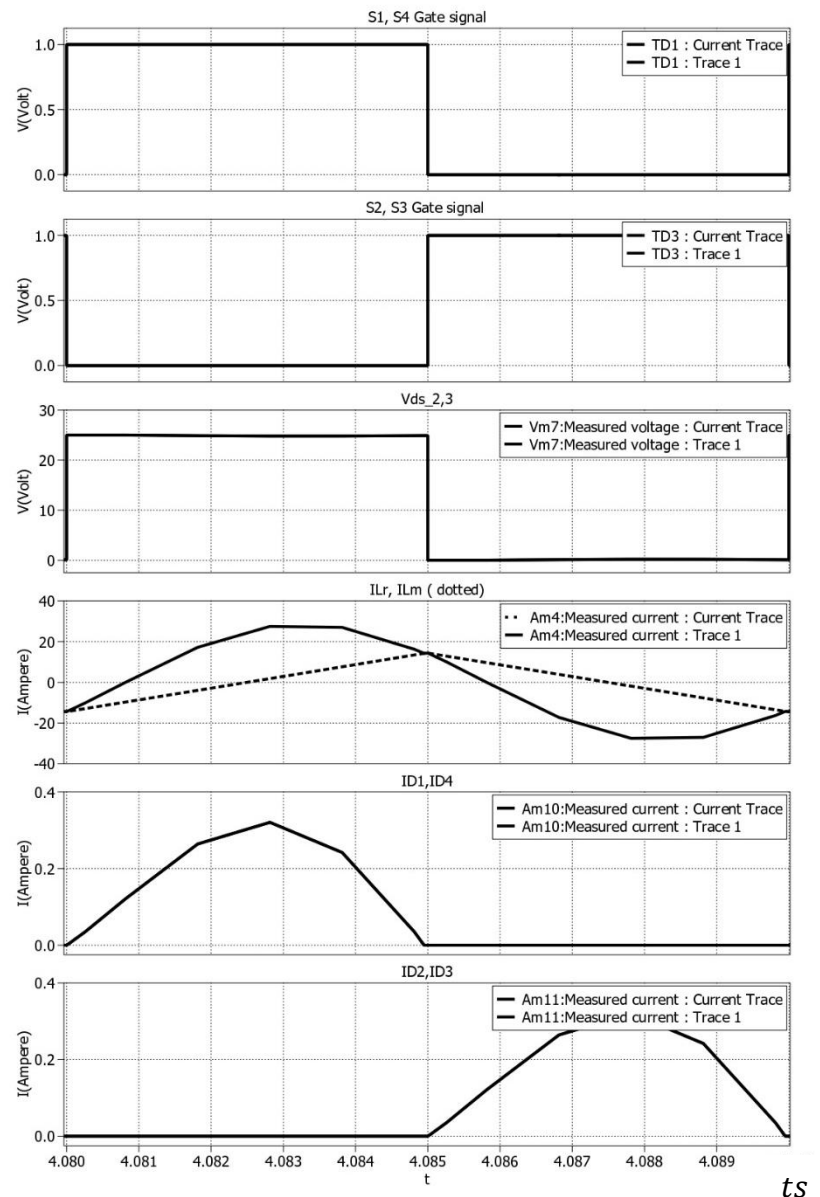


Figure 3.1 At resonance

### 3.1.1 Analytical Expression for the Magnetising Current at Resonance:

For  $0 < t < \frac{ts}{2}$  ( $ts$  is the time period of switching signal)

$$I_{Lm}(t) = i_{Lm}(0) + \frac{1}{L_m} \int_0^t v(t) dt \quad (17)$$

Where  $I_{Lm}(t)$  is Magnetising current

$v(t)$  is the transformer primary voltage which comes across the  $L_m$

$$v(t) = nV_0 \quad (18)$$

Where

$V_0$  is the transformer secondary voltage

$n$  is the turn ratio

$$I_{Lm}(t) = i_{Lm}(0) + \frac{1}{L_m} \int_0^t nV_0 dt$$

$$I_{Lm}(t) = i_{Lm}(0) + \frac{nV_0 t}{L_m} \quad (19)$$

For  $t = \frac{t_r}{4}$  (where  $t_r$  is the resonance time period)

$$I_{Lm}(t) = 0$$

$$0 = i_{Lm}(0) + \frac{nV_0 \cdot \frac{t_r}{4}}{L_m}$$

$$-\frac{nV_0 t_r}{4L_m} = i_{Lm}(0) \quad (20)$$

From (19) & (20)

$$I_{Lm(t)} = \frac{nV_0 t}{L_m} - \frac{nV_0 t_r}{4L_m} \quad (21)$$

From  $\frac{t_s}{2} < t < [t_s]$

$$I_{Lm(t)} = - \left[ \frac{nV_0}{L_m} \left( t - \frac{t_s}{2} \right) - \frac{nV_0}{4L_m f_r} \right]$$

$$I_{Lm(t)} = - \frac{nV_0}{L_m} \left( t - \frac{t_s}{2} \right) + \frac{nV_0}{4L_m f_r} \quad (22)$$

### 3.1.2 Analytical Expression for Transformer Primary Current

at resonance:

From  $0 < t < \frac{t_s}{2}$

$$I_{pri}(t) = I_{peak,pri} \sin \omega_r t \quad (23)$$

Where

$I_{pri}(t)$  is the transformer primary current

$I_{peak,pri}$  is the peak value of transformer primary current

Since,

$$I_{peak,sec} = \frac{\pi}{2} I_0$$

$I_0$  is the output current

$$I_{pri}(t) = \frac{\pi}{2n} I_0 \sin \omega_r t$$

$$I_{pri}(t) = \frac{\pi}{2n} I_0 \sin \omega_r t \quad (24)$$

∴ Where  $\omega_r = 2\pi f_r$

From  $\frac{t_s}{2} < t < t_s$

$$I_{pri}(t) = -\frac{\pi}{2n} I_0 \sin \omega_r t \quad (25)$$

### 3.2 Below Resonance

1. Half of the cycle contains the power delivery operation
2. ZVS achieved
3. DCM on secondary (Discontinuous conduction mode)
4. Rectifier are soft switched.

In this region the switching frequency is lower than the  $f_r$  but still higher than the  $f_p$  (resonance of  $L_m$ ,  $L_r$  and  $C_r$ ). The lower  $f_{r2}$  resonance frequency varies with the load change.

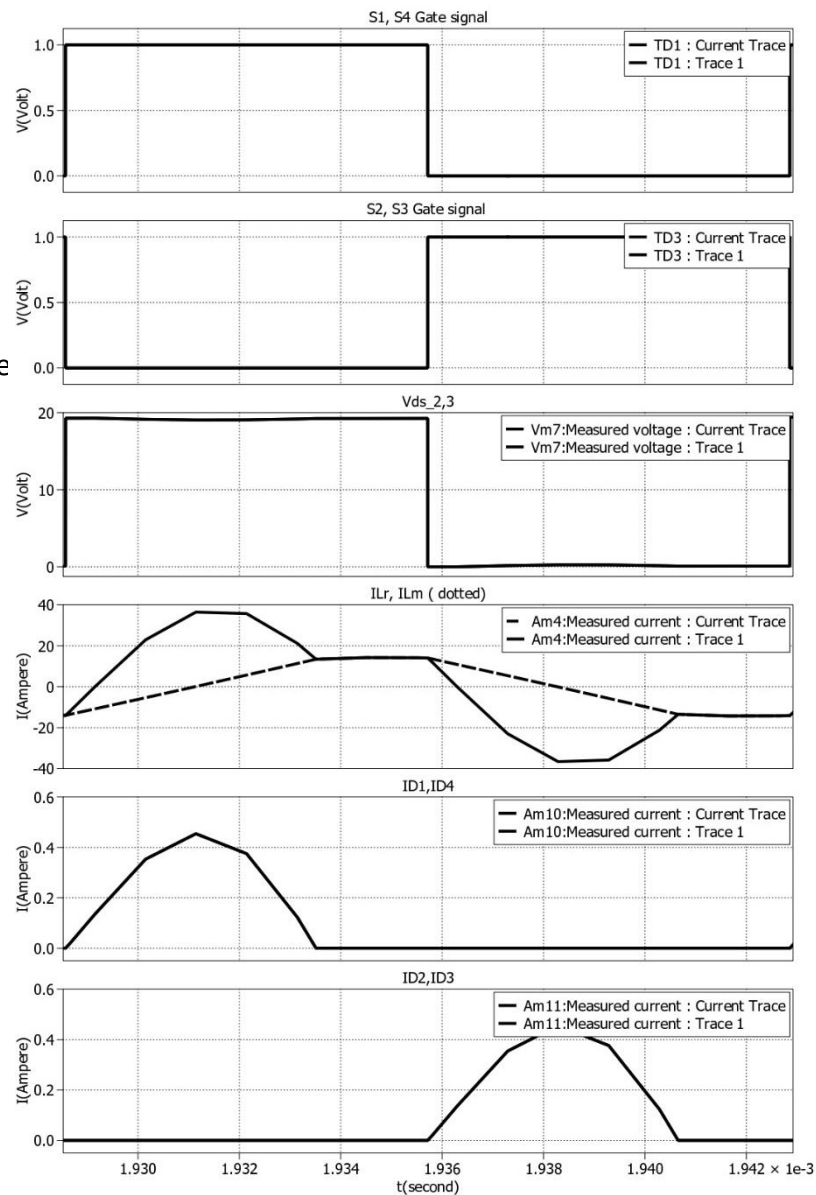


Figure 3.2 Below resonance



This interval can be divided into two phases:

- a. One when  $L_r$  resonates with  $C_r$  only while  $L_m$  is clamped by the out put voltage.
- b. Second when  $L_m$  participates in resonance when  $I_{L_r}$  (resonance current) becomes equal to the  $I_{L_m}$  (magnetising current) which is the beginning of the second phase

One benefit of LLC filter is that the resonance frequency becomes the function of load, having a range between  $f_p < f_0 < f_r$ . At short circuit load  $R_{ac} \ll L_m$  and the resonance frequency will be  $f_0 = f_r$  [2].

$$f_r = \frac{1}{2\pi\sqrt{L_r \cdot C_r}} \quad (26)$$

At no load  $R_{ac} \gg L_m$  and hence the resonance frequency will be  $f_0 = f_p$  [2].

$$f_p = \frac{1}{2\pi\sqrt{(L_m + L_r)C_r}} \quad (27)$$

LLC resonant filter is designed at  $f_s = f_0 = f_r$ , at which we achieve the unity gain of the resonant tank. Which is best operating point.

### 3.2.1 Analytical Expression for the Magnetising Current Below Resonance:

For  $0 < t < \frac{T_r}{2}$

$$I_{Lm}(t) = i_{Lm}(0) + \frac{1}{L_m} \int_0^t v(t) dt \quad (28)$$

$$v(r) = nV_0$$

$$I_{Lm(t)} = i_{Lm} + (0) \frac{1}{L_m} \int_0^t nV_0 dt$$

$$I_{Lm(t)} = i_{Lm}(0) + \frac{nV_0 t}{L_m} \quad (29)$$

For  $t = \frac{t_r}{4}$

$$I_{Lm(t)} = 0$$

$$0 = i_{Lm}(0) + \frac{nV_0 \cdot \frac{t_r}{4}}{L_m}$$

$$-\frac{nV_0 t}{4L_m} = i_{Lm}(0) \quad (30)$$

From (29) & (30);

$$I_{Lm(t)} = \frac{nV_0 t}{L_m} - \frac{nV_0 t_r}{4L_m} \quad (31)$$

from  $\frac{t_r}{2}$  to  $\left[\frac{t_s}{2}\right]$

$$I_{Lm(t)} = \frac{nV_0}{4L_m f_r} \quad (32)$$

From  $\frac{t_s}{2} < t < \left[\frac{t_s}{2} + \frac{t_r}{2}\right]$

$$I_{Lm(t)} = -\left[\frac{nV_0}{L_m} \left(t - \frac{t_s}{2}\right) - \frac{nV_0}{4L_m f_r}\right]$$

$$I_{Lm(t)} = -\frac{nV_0}{L_m} \left(t - \frac{t_s}{2}\right) + \frac{nV_0}{4L_m f_r} \quad (33)$$

From  $\frac{t_s}{2} + \frac{t_r}{2} < t < t_s$

$$I_{Lm}(t) = -\frac{nV_0}{4L_m f_r} \quad (34)$$

### 3.2.2 Analytical Expression for Transformer Primary Current Below Resonance:

From  $0 < t < \frac{t_r}{2}$

In this case, the rectifier diodes will no more conduct from 0 to  $\frac{t_s}{2}$  but those will conduct from 0 to  $\frac{t_r}{2}$ .

Therefore we will multiply the  $I_0$  with a factor  $\frac{f_r}{f_r}$  [3].

$$I_{pri}(t) = \frac{\pi}{2n} I_0 \times \left(\frac{f_r}{f_r}\right) \sin \omega_r t \quad (35)$$

From  $\frac{t_r}{2} < t < \frac{t_s}{2}$

$$I_{pri}(t) = 0 \quad (36)$$

From  $\frac{t_s}{2} < t < \left[\frac{t_s}{2} + \frac{t_r}{2}\right]$

$$I_{pri}(t) = -\frac{\pi}{2n} I_0 \left(\frac{f_r}{f_r}\right) \sin \omega_r t \quad (37)$$

From  $\left[\frac{t_s}{2} + \frac{t_r}{2}\right] < t < t_s$

$$I_{pri}(t) = 0 \quad (38)$$

### 3.3 Above Resonance

- 1.
2. Partial power delivery cycle
3. ZVS is achieved .
4. CMM on secondary

(continuous conduction mode)

5. Rectifier not soft switched
6. Increased turn off losses in primary side MOSFETs

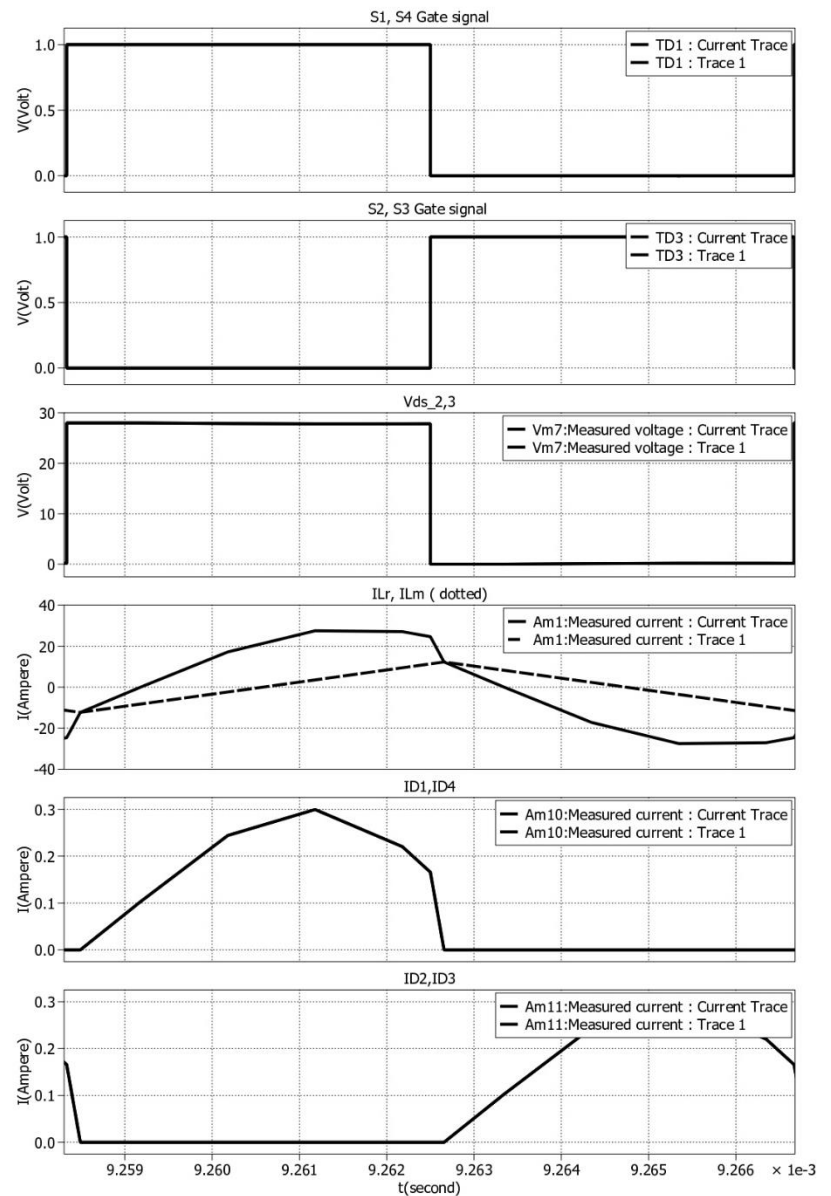


Figure 3.3 Above resonance

Analytical expression for the magnetising current above resonance:

For  $0 < t < \frac{t_s}{2}$

$$I_{Lm}(t) = i_{Lm}(0) + \frac{1}{L_m} \int_0^t v(t) dt$$

$$v(t) = nV_0$$

$$I_{Lm}(t) = i_{Lm}(0) + \frac{1}{L_m} \int_0^t nV_0 dt$$

$$I_{Lm}(t) = i_{Lm}(0) + \frac{nV_0 t}{L_m} \quad (39)$$

For  $t = \frac{t_r}{4}$

$$I_{Lm}(t) = 0$$

$$0 = i_{Lm}(0) + \frac{nV_0 \cdot \frac{t_r}{4}}{L_m}$$

$$-\frac{nV_0 t}{4L_m} = i_{Lm}(0) \quad (40)$$

From (39) & (40);

$$I_{Lm}(t) = \frac{nV_0 t}{L_m} - \frac{nV_0 t_r}{4L_m} \quad (41)$$

From  $\frac{t_s}{2} < t < [t_s]$

$$I_{Lm}(t) = -\left[ \frac{nV_0}{L_m} \left( t - \frac{t_s}{2} \right) - \frac{nV_0}{4L_m f_r} \right]$$

$$I_{Lm}(t) = -\frac{nV_0}{L_m} \left( t - \frac{t_s}{2} \right) + \frac{nV_0}{4L_m f_r} \quad (41)$$

### 3.3.1 Analytical expression for transformer primary current above resonance:

From  $0 < t < \frac{t_s}{2}$

Output current of rectifier is

$$I_0 = \frac{2}{t_s} \int_0^{\frac{t_s}{2}} I_{peak} \sin \omega_r t. dt \quad (42)$$

$\therefore$  from 0 to  $\frac{t_s}{2}$

$$I_{peak,sec} = \frac{I_0 t_s}{2 \int_0^{\frac{t_s}{2}} \sin \omega_r t. dt} \quad (43)$$

$$I_{pri}(t) = \frac{I_{peak,sec}}{n} \sin \omega_r t$$

From  $\frac{t_s}{2} < t < t_s$

$$I_{pri}(t) = -\frac{I_{peak,sec}}{n} \sin \omega_r t \quad (44)$$

## 3.4 Operation Principle

The operation of a half bridge LLC converter is explain here (one leg of full bridge). Same principle is applied for the second leg of full bridge LLC converter. The circuit diagram for half bridge LLC converter is shown in figure 3.4. The waveforms for this LLC converter are shown in figure 3.5 [3].

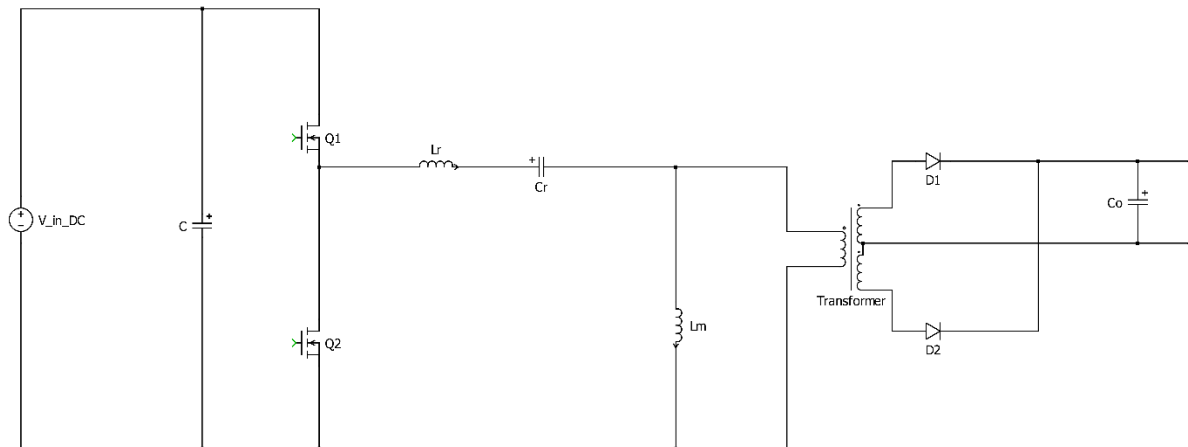


Figure 3.4 Half Bridge LLC Converter

### 3.4.1 Power Delivery Period 1 ( $t_0 \rightarrow t_4$ )

During this period

$Q_1$  is turned ON

$Q_2$  is turned OFF

(i)  $t_0 \rightarrow t_1$

The resonance current  $I_{RT}$  (fig 3.5) flows by the  $Q_1$  while magnetizing current  $I_{Lm}$  flows through both  $Q_1$  & diode of  $Q_1$ .

Since  $Q_1$  is not turned on fully, Power is transmitted from primary to secondary side by  $I_{TR} (I_{RT} > 0)$ .

$I_{Lm}$  increases linearly because it is proportional to output voltage.

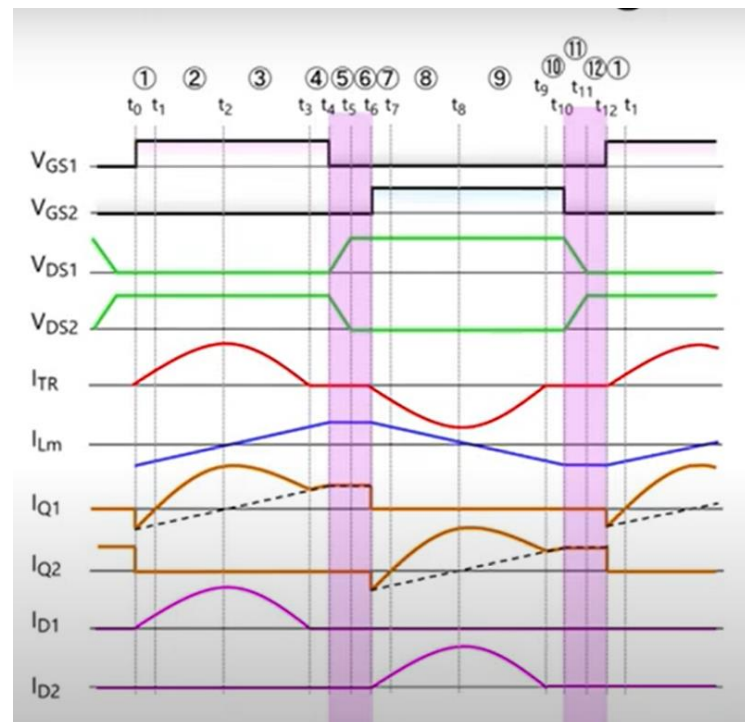


Figure 3.5 Operating Cycles

(ii)  $t_1 \rightarrow t_2$

Now  $Q_1$  is completely functioning.

Resonance current at primary reaches to peak at  $t_2$ .

The polarity of  $I_m$  changes from peak negative to zero.

Both  $I_m$  &  $I_{TR}$  flow by  $Q_1$ .

(iii)  $t_2 \rightarrow t_3$

$I_m$  reaches zero to near peak positive.

$I_{TR}$  reaches peak to zero.

At  $t_3$ ,  $I_{TR}$  becomes zero, so power transmission from primary to secondary stops.

(iv)  $t_3 \rightarrow t_4$

Resonance current is zero. Only magnetizing current flows.

$I_m$  reaches at positive peak at time  $t_4$ .

### 3.4.2 Soft Switching Period for Q1 ( $t_4 \rightarrow t_6$ )

Both  $Q_1$  &  $Q_2$  are OFF.

(i)  $t_4$  To  $t_5$

Only magnetizing  $I_m$  flows continuously in this period.

Magnetizing current charges  $C_{Q1}$  which is the parasitic capacitance of  $Q_1$  and discharges  $C_{Q2}$ .

At the end of this period,  $C_{Q1}$  is charge to almost  $V_{in}$  and  $C_{Q2}$  is discharge to almost zero.

The discharging of  $C_{Q2}$  to zero volts satisfy the zero voltage switching, (soft switching one requirement to turn on  $Q_2$ ).

(ii)  $t_5$  To  $t_6$

The charging and discharging of  $C_{Q1}$  &  $C_{Q2}$  by  $I_m$  is completed.

However, magnetizing current continues to flow though it will flow through the body diode of  $Q_2$  ( $D_{Q2}$ ).

Consequently, the  $V_{DS2}$  will be nearly zero volts and thus ready for soft switching.

### 3.4.3 Power Delivery Period 2 ( $t_6 \rightarrow t_{10}$ )

$Q_1$  Is OFF

$Q_2$  is ON

(i)  $t_6$  To  $t_7$

$I_m$  flows through  $Q_2$  &  $D_{Q2}$ . since  $Q_2$  is not completely turned on and  $I_{TR}$  flows from  $Q_2$  only. Its flow is because of suspended power in  $C_r$

(ii)  $t_7$  To  $t_8$

$Q_2$  Is fully functioning



No current flows by body diode.

$I_{Lm}$  reaches positive peak to zero linearly (decreases).

$I_{TR}$  reaches zero to negative peak.

Its power transmission cycle.

(iii)  $t_6$  To  $t_9$

Resonance current reaches to zero. (power transmission stops)

$I_{Lm}$  Reaches zero to negative peak.

(iv)  $t_9$  To  $t_{10}$

only magnetizing current flows and resonant does not.

$I_m$  Reaches to the peak at  $t_{10}$

#### 3.4.4 Soft Switching Period for Q1 ( $t_{10} \rightarrow t_{12}$ )

$Q_1$  is OFF

$Q_2$  is OFF

(i)  $t_{10}$  To  $t_{11}$

$I_m$  Charges  $C_{Q2}$  and discharges  $C_{Q1}$  and at the time  $t_{11}$ ,  $C_{Q2}$  is fully charged and  $C_{Q1}$  is fully discharged ( $V_{Qd1} \approx 0$ ) so  $Q_1$  is ready for soft switching.

(ii)  $t_{11}$  To  $t_{12}$

Charging and discharging of  $C_{Q1}$  and  $C_{Q2}$  stops &  $I_m$  flows through body diode of  $Q_1$  (because  $V_{Qd1} \approx 0$ ) which is ready for soft switching.

## 4 Design Steps

In the design steps the first step is the selecting the Q values. First thing is to plot the curves for different Q values for some initial m values that can be optimized in second iteration or third iteration of the design steps.

The transformer turn ratio and the minimum and maximum voltage gains of the resonant tank are calculated first to determine the appropriate Q value to produce the required gain range of tank.

In our design we have following data

*Table 4-1 Design Specifications*

Output voltage	2000V
Input voltage	15-30V (25V nominal)
Output power	400W
Resonant frequency	100kHz

$$M_{nom} = 1 \quad (45)$$

Where  $M_{nom}$  is the nominal tank gain which is unity (in case of resonance)

$$\frac{N_p}{N_s} = \frac{V_{in\_nom}}{V_{out}} \cdot M_{nom} = 0.0125 \quad (46)$$

$$M_{max} = \frac{V_{in\_nom}}{V_{in\_min}} \cdot M_{nom} = 1.667 \quad (47)$$

$$M_{min} = \frac{V_{in\_nom}}{V_{in\_max}} \cdot M_{nom} = 0.833 \quad (48)$$

Where  $M_{max}$  is the maximum required tank gain.

$M_{min}$  is the minimum required tank gain

#### 4.1 Selecting The $Q_{max}$ Value

If we keep the m factor fix and we can see the impact of Q factor on the gain. The higher the Q value the lower the gain of resonant tank. The peaks of the curves define the boundary between capacitive and inductive region. We only operate in the inductive region since inductive region provides the ZVS. In capacitive region the current will lead the voltage and reverse current can flow in the MOSFET. It makes noise and reverse recovery losses. It might create the high current spikes as well. The value of  $Q_{max}$  is always set at the max load point to fulfil the operating condition when load changes.

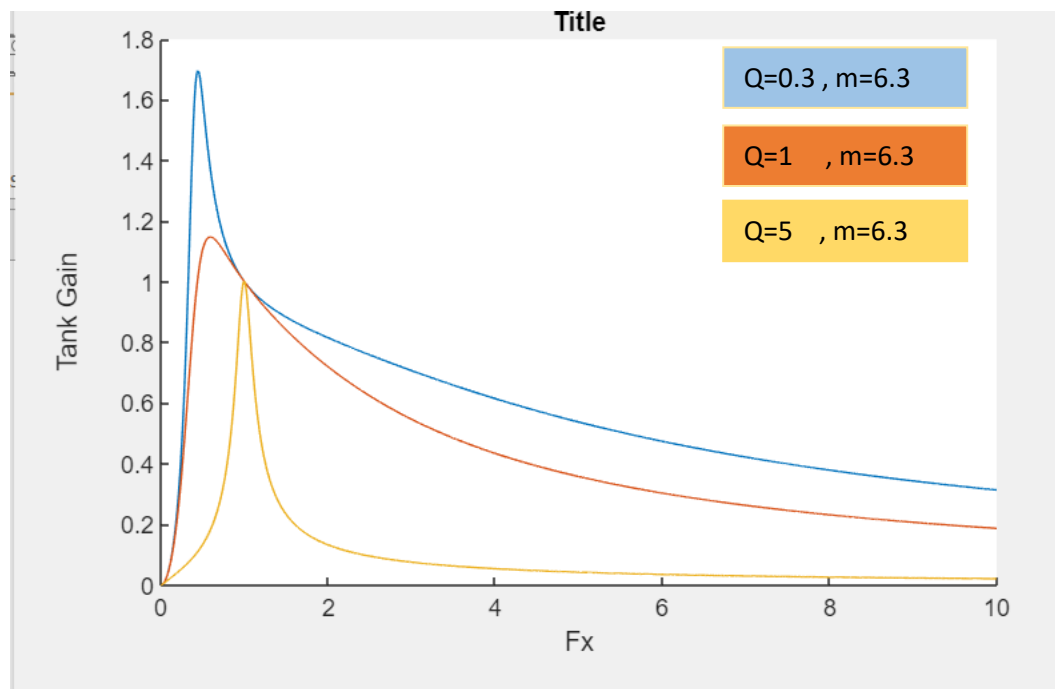


Figure 4.1 Tank gain with varying Q value and fixed m value

We can decrease Q value in order to increase the resonant tank gain but frequency modulation range increases. Therefore, we cannot rely on Q value to change the gain. For our design we have chosen a moderate value  $Q_{max} = 0.4$ .

## 4.2 Selecting The m Value

Lower value of  $m$  cause higher boost in gain and with narrow frequency range. It is more flexible regulation but as the magnetizing inductance decreases. It causes higher circulating current and lower efficiency. As shown figure 4.2.

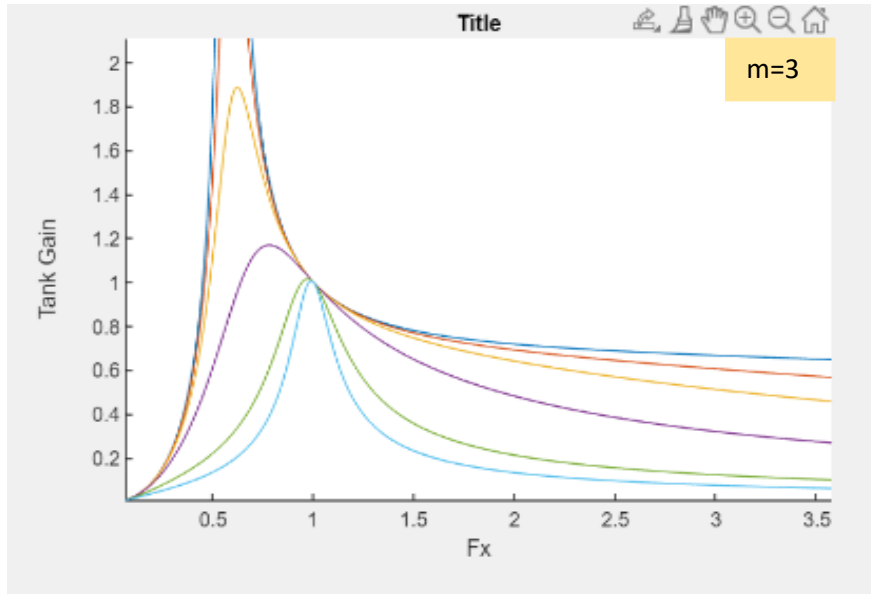


Figure 4.2 Tank gain at low value of  $m$

High  $m$  value gives magnetizing inductance causing lower circulating current and higher efficiency. But gain is decreased. As shown figure 4.3.

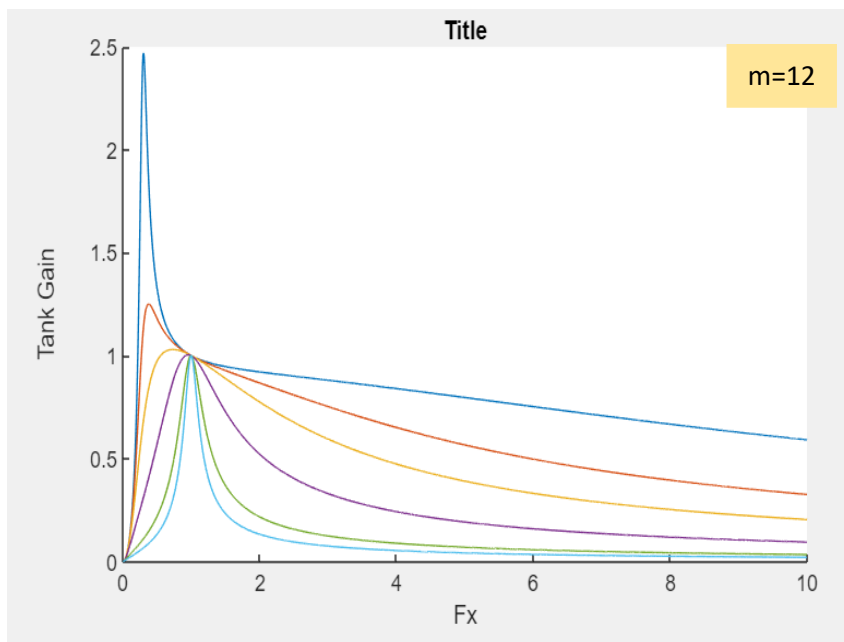


Figure 4.3 Tank gain at high value of  $m$

Therefore, a moderate  $m$  value should be selected. For our design  $m=6.3$ .

As shown figure 4.4.

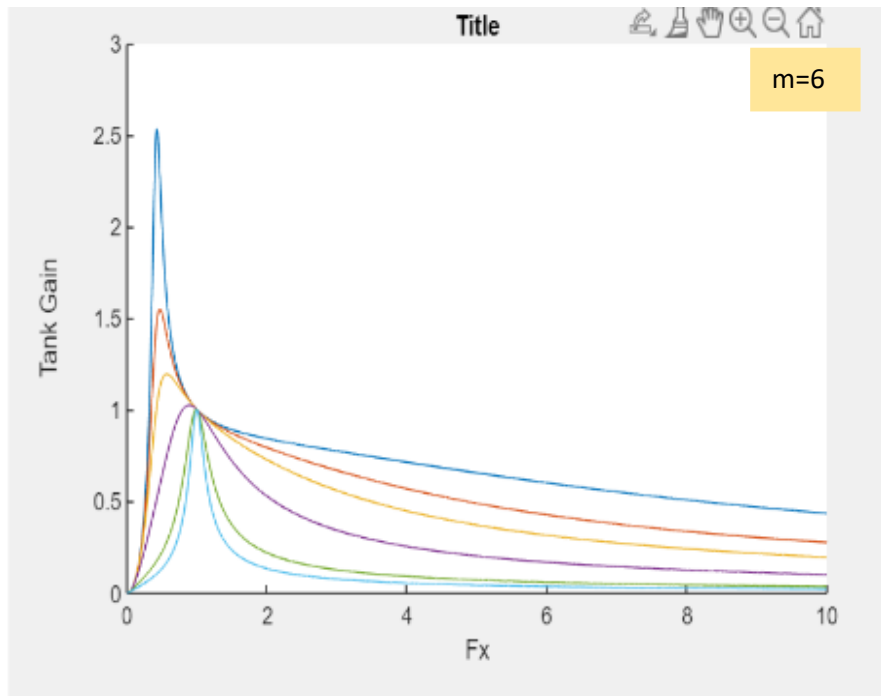


Figure 4.4 Tank gain at moderate value of  $m$

### 4.3 Finding The Minimum Normalized Switching Frequency

The main control parameter is the normalized frequency which is set by minimum value for  $Q_{max}$  and a moderate value of  $m$ . There are three zones of operation:

1. At resonance frequency  $f_s = f_r$  : In this mode the tank gain of resonant tank is Unity.
2. Above resonance frequency  $f_s > f_r$  : In this mode of operation the tank gain is lower than unity.(Buck)
3. Below resonance frequency  $f_s < f_r$  : In this mode the resonant tank gain is higher than unity.(Boost)

After selecting the  $Q_{max}$  and  $m$  value [2]

$$\frac{d}{dF_x} K(Q_{max}, m, F_{xmin})|_{Q_{max}, m} = 0 \quad (49)$$

Eq. (49) Gives  $F_{xmin} = 0.489$

#### 4.4 Required Voltage Gain vs Available Voltage Gain

As we know [2].

$$Q_{max@vmin} = Q_{max} \cdot \frac{V_{in\_min}}{V_{in\_max}} = 0.4 * \frac{15}{25} = 0.2 \quad (50)$$

$$K_{max} = K(Q_{max@vmin}, m, F_{xmin}) = 1.974 \quad (51)$$

in our case  $K_{max} > M_{max}(1.667)$  no need for turning the tank Gain .

if  $K_{max} < M_{max}$  turning the tank gain is required.

Where

$K_{max}$  is the maximum available tank gain at  $V_{in}$  minimum

From eq. (9), (10) ,(11) and (26) we will get the resonant tank parameter

Which are

Table 4-2 Resonant Tank Parameter

$L_r$	0.8063 $\mu H$
$L_m$	4.273 $\mu H$
$C_r$	3.142 $\mu F$

## 4.5 Design Flow Chat

The following flow chat is used to understand the design steps mentioned above [2].

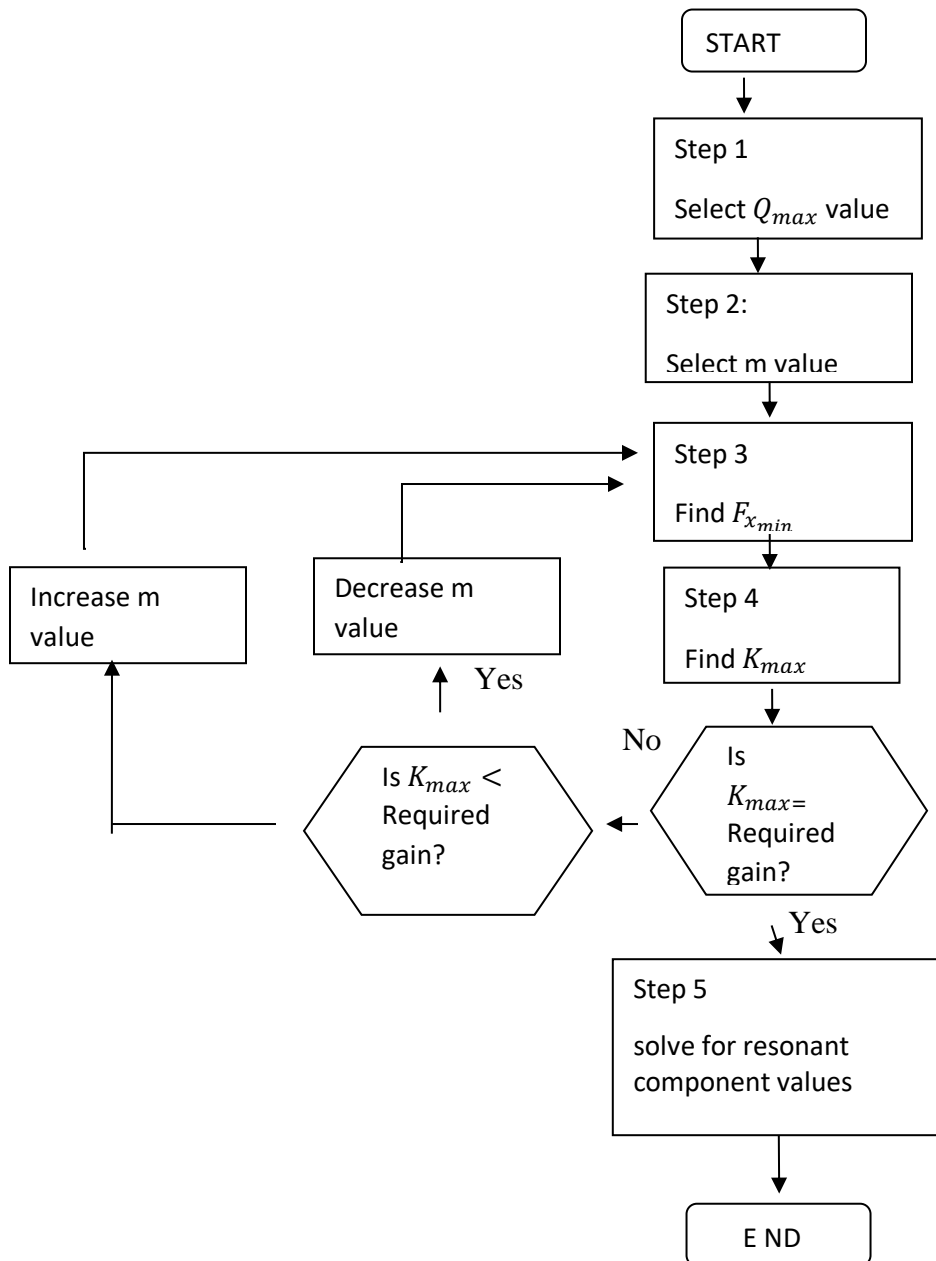


Figure 4.5 Design flow chart

## 4.6 Power Delivery Cycle

Power delivery operation, which occurs twice in a switching cycle. The difference between the resonant current and the magnetizing current passes through the transformer and rectifier to the secondary side, and power is delivered to the load.

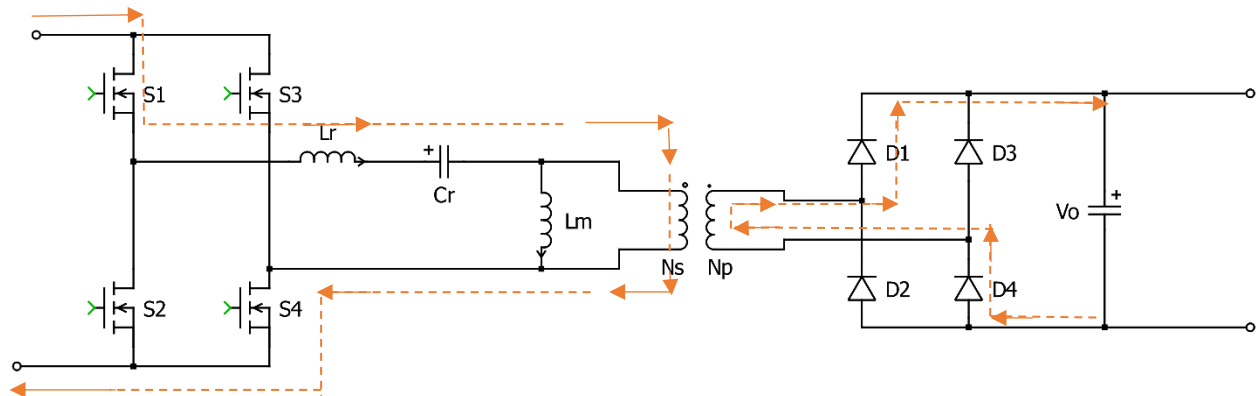


Figure 4.6 Power delivery cycle

## 4.7 Freewheeling Cycle

When the resonance current reaches the magnetizing current, freewheeling operation begins following the power delivery operation. It happens only below resonance operation when switching frequency is less than the resonance frequency, causing transformer secondary current to zero and the secondary side rectifier to disconnect.  $L_m$  enters the resonance with  $L_r$  and  $C_r$ .

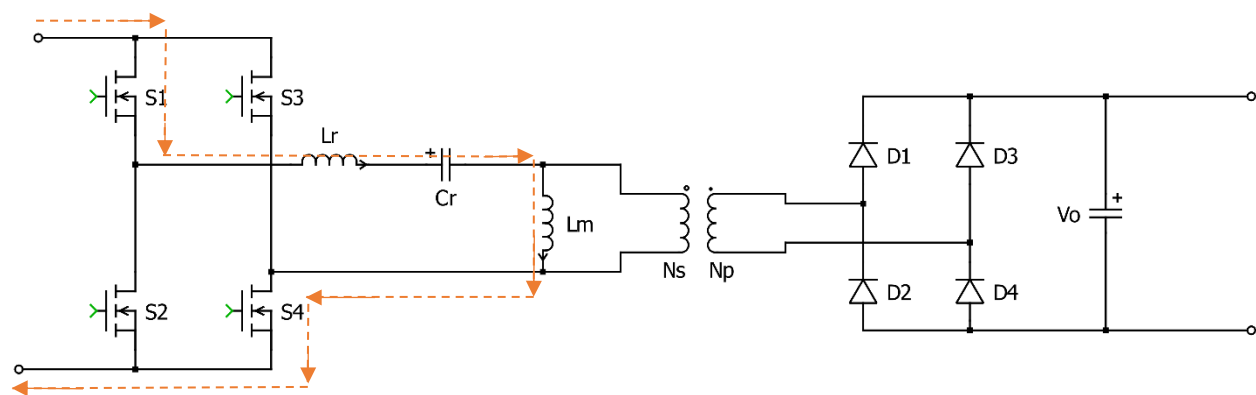


Figure 4.7 Freewheeling cycle



## 5 Transformer Design

Consider a two-winding transformer as shown in **Error! Reference source not found.**. The core has cross-sectional area  $A_c$ , mean magnetic path length  $\ell_m$ , and permeability  $\mu$ , where  $n_1, n_2$  are the turns on the primary and secondary of the transformer and  $v_1, v_2$  their respective voltages. The transformer currents are represented with  $i_1, i_2$  respectively for the primary side and secondary side. An equivalent magnetic circuit is given in **Error! Reference source not found.**

The core reluctance is

$$\mathcal{R} = \frac{\ell_m}{\mu A_c} \quad (52)$$

Since there are two windings in this example, it is necessary to determine the relative polarities of the MMF (Magnetomotive Force) generators. Ampere's law states that

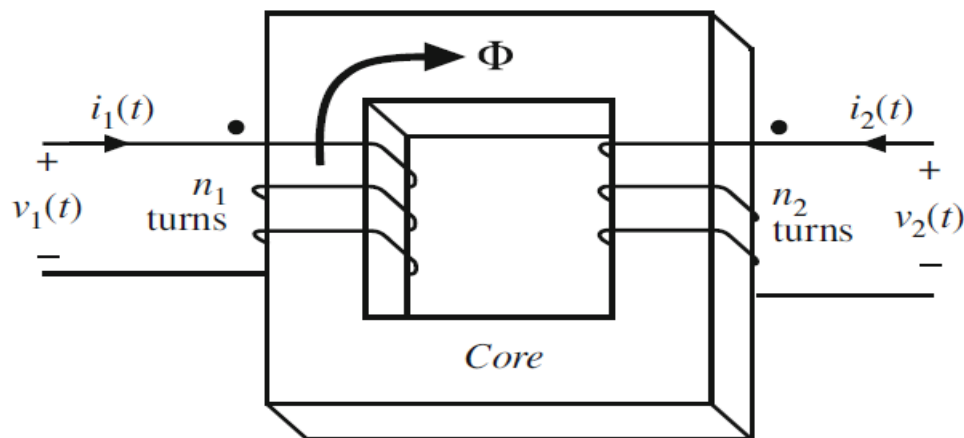


Figure 5.1 A two-winding transformer [4]

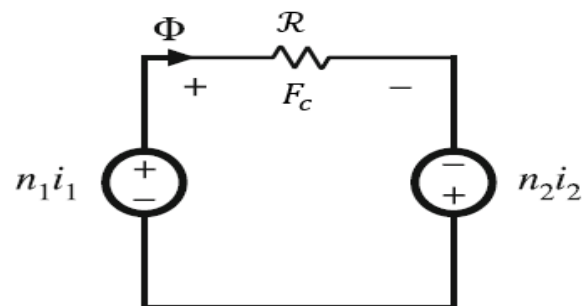


Figure 5.2 Magnetic circuit that models the two-winding transformer of Figure 5.1 [4]

$$F_c = n_1 i_1 + n_2 i_2 \quad (53)$$

where  $F_c$  the magnetomotive force 'MMF' across the core and  $\Phi(t)$  is the flux passes through the windings and the MMF generators are additive because the currents  $i_1$  and  $i_2$  pass in the same direction through the core window. Solution of **Error! Reference source not found.** yields

$$\Phi \mathcal{R} = n_1 i_1 + n_2 i_2 \quad (54)$$

This expression could also be obtained by substitution of  $F_c = \Phi \mathcal{R}$  into Eq. (53).

## 5.1 The Ideal Transformer

In the ideal transformer, the core reluctance  $\mathcal{R}$  approaches zero. This causes the core MMF  $F_c = \Phi \mathcal{R}$  also to approach zero. Eq (53) then becomes

$$0 = n_1 i_1 + n_2 i_2 \quad (55)$$

Also, by Faraday's law, we have

$$v_1(t) = n_1 \frac{d\Phi(t)}{dt} \quad (56)$$

$$v_2(t) = n_2 \frac{d\Phi(t)}{dt} \quad (57)$$

Note that  $\Phi$  is the same in both equations above: the same total flux links both windings. Elimination of  $\Phi$  leads to

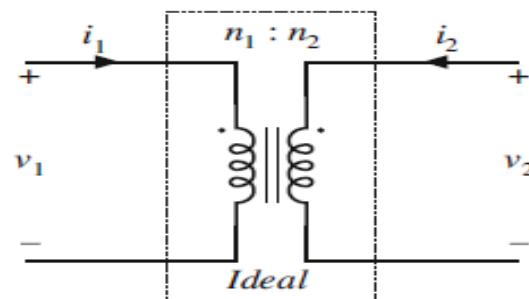


Figure 5.3 Ideal transformer symbol [4]

$$\frac{d\Phi}{dt} = \frac{v_1}{n_1} = \frac{v_2}{n_2} \quad (58)$$

Eq (55) and (56) are the equations of the ideal transformer:

$$\frac{v_1}{n_1} = \frac{v_2}{n_2} \quad \text{and} \quad n_1 i_1 + n_2 i_2 = 0 \quad (59)$$

The ideal transformer symbol of Figure 5.3 is defined by Eq (59).

### 5.1.1 The Magnetizing Inductance

For the actual case in which the core reluctance  $\mathcal{R}$  is nonzero, we have

$$\Phi \mathcal{R} = n_1 i_1 + n_2 i_2 \quad \text{with} \quad v_1(t) = n_1 \frac{d\Phi(t)}{dt} \quad (59)$$

Elimination of  $\Phi$  yields

$$v_1 = \frac{n_1^2}{\mathcal{R}} \frac{d}{dt} \left[ i_1 + \frac{n_2}{n_1} i_2 \right] \quad (1)$$

This equation is of the form

$$v_1 = L_M \frac{di_M}{dt} \quad (2)$$

From Eq (60)

$$L_M = \frac{n_1^2}{\mathcal{R}}$$

$$i_M = i_1 + \frac{n_2}{n_1} i_2 \quad (3)$$

$L_M$  is the magnetizing inductance and  $i_M$  magnetizing current, referred to the primary winding. An equivalent circuit is illustrated in Figure 5.4

The magnetizing inductance models the magnetization of the core material. It is a real, physical inductor, which exhibits saturation and hysteresis. All physical transformers must contain magnetizing inductance. For example, suppose that we disconnect the secondary winding. We are then left with a single winding on a magnetic core—an inductor. Indeed, the equivalent circuit of Figure 5.4

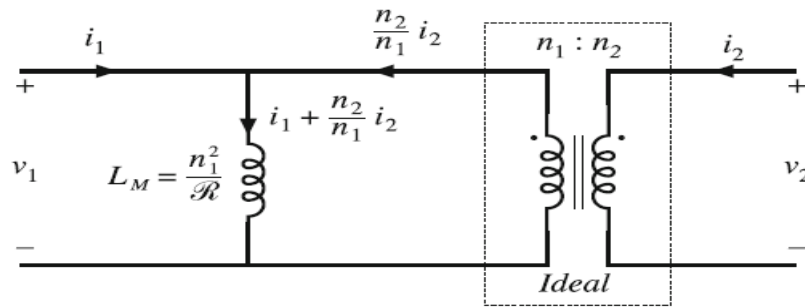


Figure 5.4 Transformer model including magnetizing inductance [4]

predicts this behavior, via the magnetizing inductance. The magnetizing current causes the ratio of the winding currents to differ from the turn's ratio.

The transformer saturates when the core flux density  $B(t)$  exceeds the saturation flux density  $B_{sat}$ . When the transformer saturates, the magnetizing current  $i_M(t)$  becomes large, the impedance of the magnetizing inductance becomes small, and the transformer windings become short circuits. It should be noted that large winding currents  $i_1(t)$  and  $i_2(t)$  do not necessarily cause saturation: if these currents obey Eq. (55), then the magnetizing current is zero and there is no net magnetization of the core. Rather, saturation of a transformer is a function of the applied volt-seconds. The magnetizing current is given by

$$i_M = \frac{1}{L_M} \int v_1(t) dt \quad (4)$$

Alternatively, Eq. (63) can be expressed in terms of the core flux density  $B(t)$  as

$$B(t) = \frac{1}{n_1 A_c} \int v_1(t) dt \quad (5)$$

The flux density and magnetizing current will become large enough to saturate the core when the applied volt-seconds  $\lambda_1$  is too large, where  $\lambda_1$  is defined for a periodic ac voltage waveform as

$$\lambda_1 = \int v_1(t) dt \quad (6)$$

The limits are chosen such that the integral is taken over the positive portion of the applied periodic voltage waveform.

To fix a saturating transformer, the flux density should be decreased by increasing the number of turns, or by increasing the core cross-sectional area  $A_c$ . Adding an air gap has no effect on saturation of conventional transformers, since it does not modify Eq. (64). An air gap simply makes the transformer less ideal, by decreasing  $L_M$  and increasing  $i_M(t)$  without changing  $B(t)$ . Saturation mechanisms in transformers differ from those of inductors because transformer saturation is determined by the applied winding voltage waveforms, rather than the applied winding currents.

### 5.1.2 Leakage Inductances

In practice, there is some flux which links one winding but not the other, by “leaking” into the air or by some other mechanism. As illustrated in **Error! Reference source not found.**, this flux leads to leakage inductance, i.e., additional effective inductances that are in series with the windings. A topologically equivalent structure is illustrated in **Error! Reference source not found.b**, in which the leakage fluxes  $\Phi_{\ell_1}$  and  $\Phi_{\ell_2}$  are shown explicitly as separate inductors.

**Error! Reference source not found.** illustrates a transformer electrical equivalent circuit model, including series inductors  $L_{\ell_1}$  and  $L_{\ell_2}$  which model the leakage inductances. These leakage inductances cause the terminal voltage ratio  $v_2(t)/v_1(t)$  to differ from the ideal turn’s ratio  $n_2/n_1$ . In general, the terminal equations of a two-winding transformer can be written.

$$\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} \\ L_{12} & L_{22} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix} \quad (7)$$

The quantity  $L_{12}$  is called the mutual inductance, and is given by eq (67)

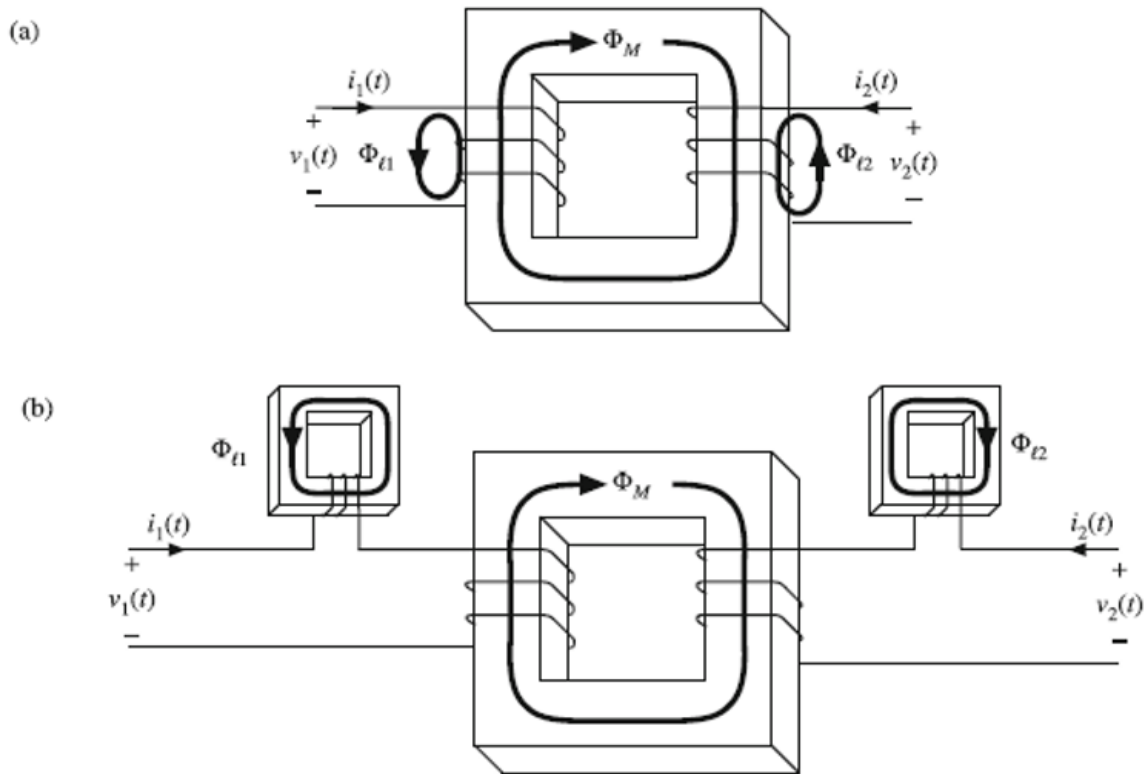


Figure 5.5 Leakage flux: (a) transformer geometry, (b) an equivalent system [2]

$$L_{12} = \frac{n_1 n_2}{\mathcal{R}} = \frac{n_2}{n_1} L_M \quad (8)$$

The quantities  $L_{11}$  and  $L_{22}$  are called the primary and secondary self-inductances, given by

$$L_{11} = L_{\ell 1} + \frac{n_2}{n_1} L_{12} \quad (9)$$

$$L_{22} = L_{\ell 2} + \frac{n_2}{n_1} L_{12} \quad (10)$$

Note that Eq. (66) does not explicitly identify the physical turns ratio  $n_2/n_1$ . Rather, Eq. (66) expresses the transformer behavior as a function of electrical quantities alone. Equation (66) can be used, however, to define the effective turns ratio

$$n_e = \sqrt{\frac{L_{22}}{L_{11}}} \quad (11)$$

and the coupling coefficient

$$k = \frac{L_{12}}{\sqrt{L_{11} L_{22}}} \quad (12)$$

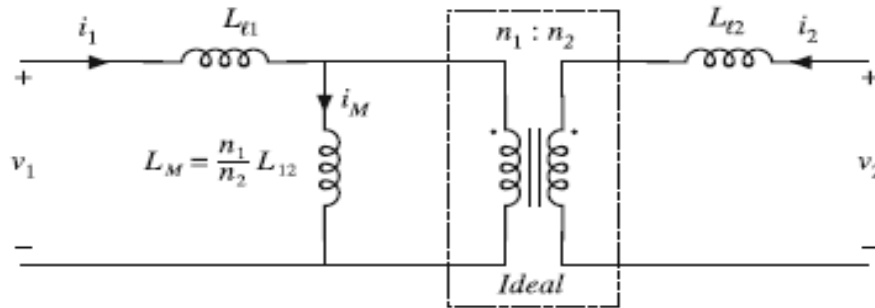


Figure 5.6 Two-winding transformer equivalent circuit [4]

The coupling coefficient  $k$  lies in the range  $0 \leq k \leq 1$  and is a measure of the degree of magnetic coupling between the primary and secondary windings. In a transformer with perfect coupling, the leakage inductances  $L_{\ell 1}$  and  $L_{\ell 2}$  are zero. The coupling coefficient  $k$  is then equal to 1. Construction of low-voltage transformers having coupling coefficients more than 0.99 is quite feasible. When the coupling coefficient is close to 1, then the effective turns ratio  $ne$  is approximately equal to the physical turn's ratio  $n_2/n_1$  [4].

## 5.2 Turns Per Volt Calculation

For designing a transformer, we need certain number of turns on each side for a specific rating transformer. Voltages on each side have direct relation with number of turns. So we are interested in finding voltage per turn and for designing, turns per voltage. This is obtained from basic voltage equation of transformer:

$$E = 4.44fB_m A_i \quad (72)$$

This equation is derived from basic equation. Here we are going to derive this.

As we know that emf induced is given by the rate of change of flux:

$$e = N \frac{d\phi}{d_t} \quad (73)$$

RMS value is linked with,

$$\sqrt{2}$$

With peak value i.e.

$$e = \sqrt{2}E$$

So putting this, we get:

$$E = \frac{N}{\sqrt{2}} \frac{d\phi}{dt} \quad (74)$$

$$\phi(t) = AB \sin(\omega t) \quad (75)$$

Taking derivative w.r.t 't' of above equation,

$$\frac{d\phi(t)}{dt} = \omega AB \cos(\omega t) \quad (76)$$

For maximum flux linkage,

$$\frac{d\phi(t)}{dt} = 2\pi f AB_m \quad (77)$$

And we know that:

$$\omega = 2\pi f$$

$$E = N \frac{2\pi f AB_m}{\sqrt{2}} \quad (78)$$



This is total emf induced. But we are interested in voltage per turn. So, dividing both sides by total number of turns (N).

$$E_T = 4.44f AB_m \quad (79)$$

5.3 Primary Conductor Design

5.4 Secondary Conductor Design

5.5 Core Design

## 6 Bridge and Rectifier Selection

### 6.1 Bridge Selection

There are two types of topologies (half bridge and full bridge ) are used for the conversion of DC input voltage to pulsating DC to feed the transformer. Both of the topologies are having their own advantages and disadvantages. In this design of LLC converter we have used full bridge converter topology.

#### 6.1.1 Half Bridge

In the half bridge topology the we use two switches (one leg) to convert input DC voltage into the pulsating output DC voltage t to feed the transformer. Half of the input voltage appears across the switches. It's a simple circuit shown in the figure 6.1.

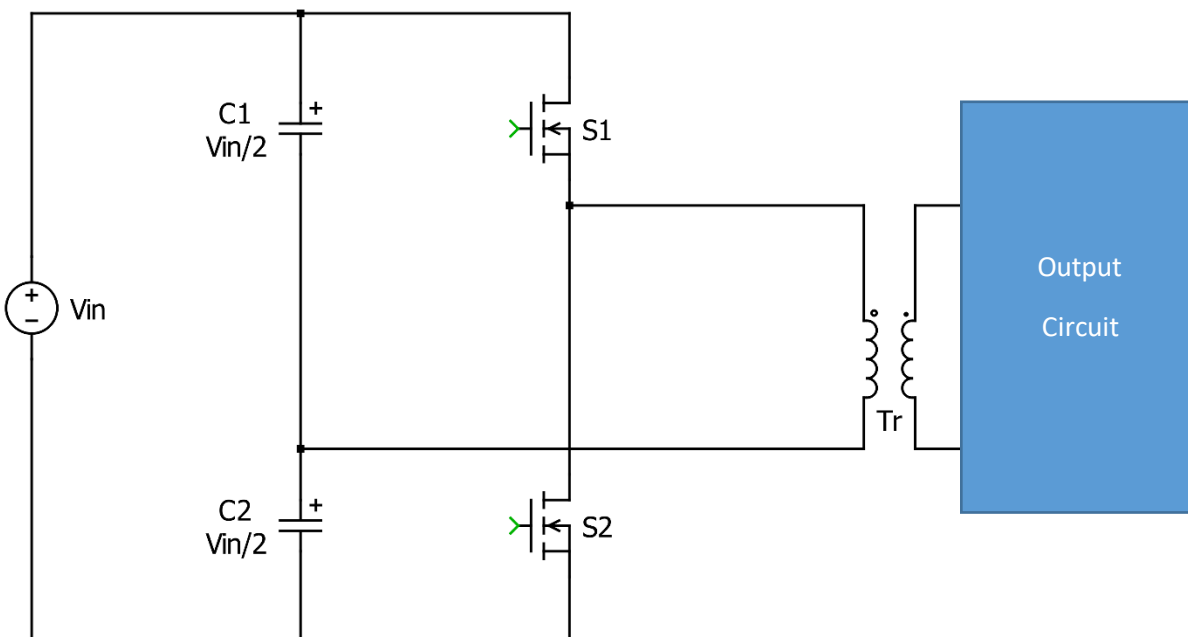


Figure 6.1 Half bridge topology

##### 6.1.1.1 Advantages of Half Bridge Topology

- It has a simple circuit.
- It's cheap since we have only two switches instead of four.
- Also only two gate drives are required.
- Lower switching losses are compared to the full bridge topology

### 6.1.1.2 Disadvantages of Half Bridge Topology

- It is functioning at  $1/2$  the input voltage where the switches are operational two times the collector current.
- Its efficiency is lower than the full bridge topology.
- This topology is not suitable for current mode control.

### 6.1.2 Full Bridge

In full bridge topology we use four switches to change the constant input DC voltage to the pulsating DC voltage. A full bridge topology is shown in the figure 6.2.

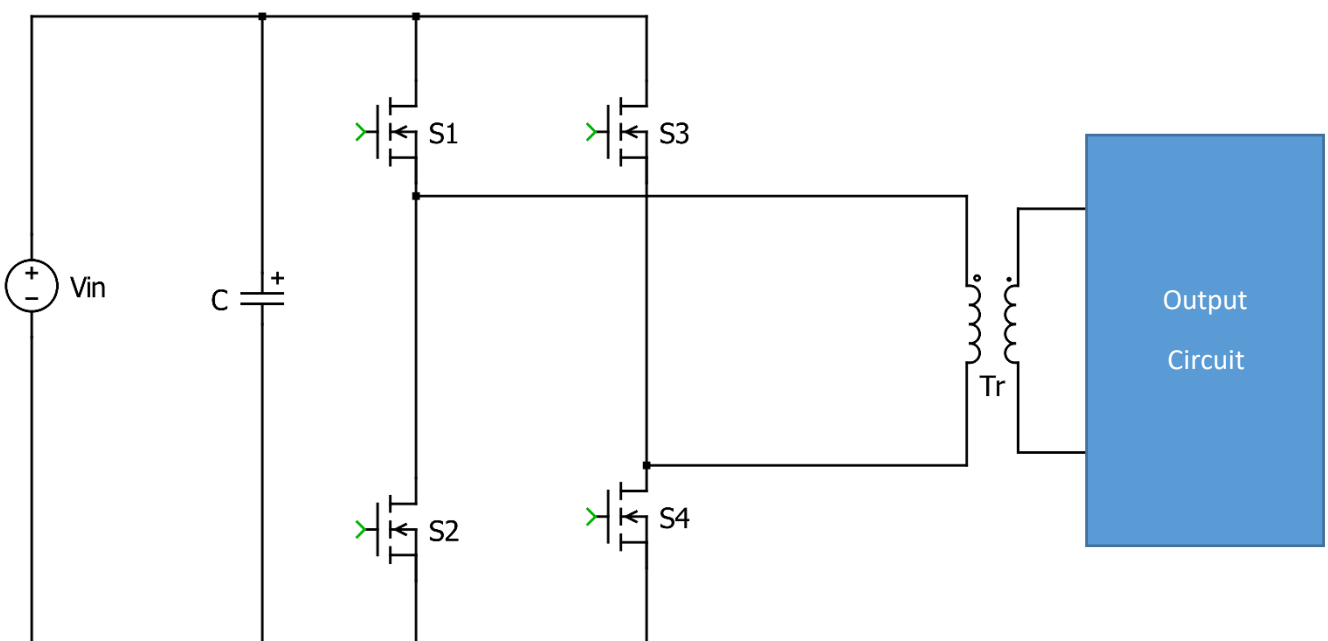


Figure 6.2 Full bridge topology

#### 6.1.2.1 Advantages of Full Bridge Topology

- It has high efficiency.
- It has lower current, since full voltage swing is applied.
- It uses four switches, switches are not more stressed which is useful at high power.

#### 6.1.2.2 Disadvantages of Full Bridge Topology

- Switches cost increases.
- Four gate drives are used for four switches.

- Switches losses increases.

### 6.1.3 Selection of Full Bridge vs Half Bridge Topology for LLC Converter

- In the case of half bridge for quite narrow voltage regulation quite void frequency range is needed.
- Full bridge resonance capacitor may not need a high rating of DC voltage.
- In case of current, full bridge has advantage of lower current since full voltage swing is applied.
- However in case of power dissipation half bridge has advantage over full bridge since power is dissipated in one switch instead of two switches.

## 6.2 Rectifier Selection

The output voltage of the transformer is feed to the rectifier which converts the AC voltage into the DC voltage. There are two types of rectifiers are used, half wave rectifier and full wave rectifier.

### 6.2.1 Halaf Wave Rectifier

Half wave rectifier allows only half cycle of the input AC voltage to pass while blocks the second half cycle. Its basic structure is shown in the figure.

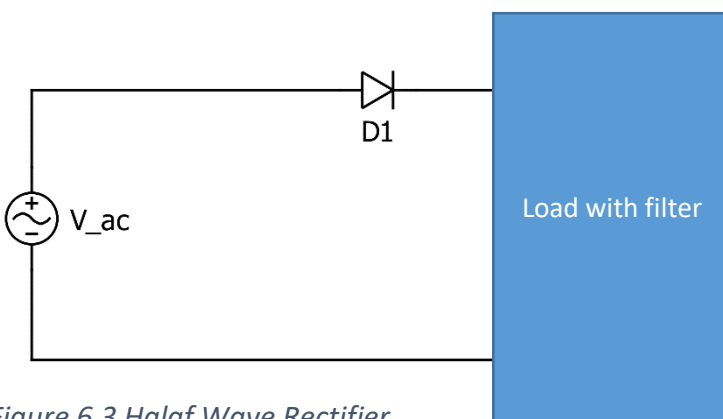


Figure 6.3 Halaf Wave Rectifier

### 6.2.1.1 Advantages of Half Wave Rectifier

- Peak inverse voltage of half wave rectifier is  $V_{out}$ .
- it's cheap, simple and easy to construct.
- voltage drop in the internal resistance will be small in half wave rectifier as compared to full wave rectifier.

### 6.2.1.2 Disadvantages of Half Wave Rectifier

- Efficiency of half wave rectifier is lower as compared to full wave rectifier.
- Ripple factor of HWR is 1.21.
- Power is delivered during only half of the cycle.

## 6.2.2 Full Wave Rectifier

Full wave rectifier converts the complete cycle of the input AC voltages into the pulsating DC voltage.

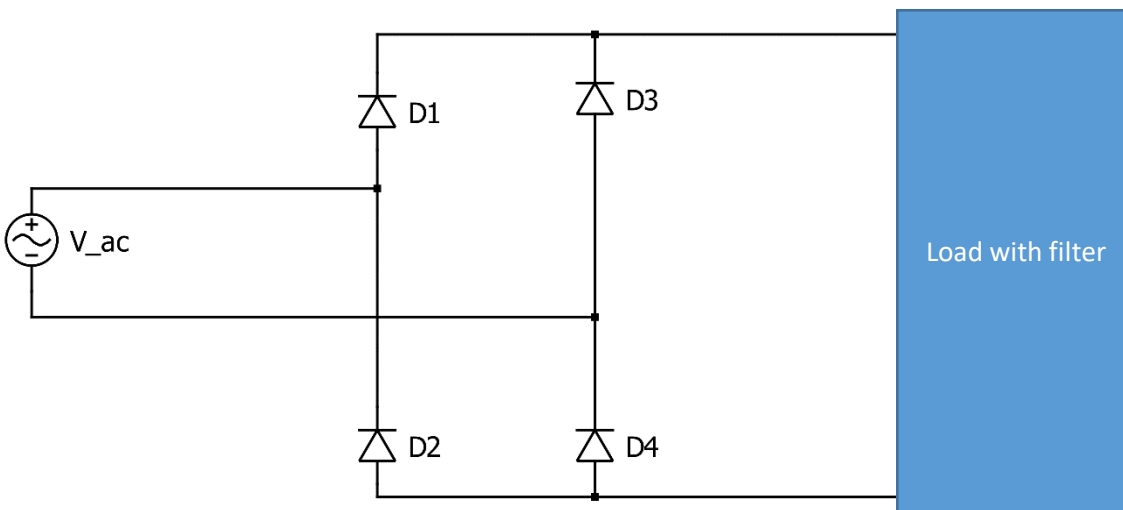


Figure 6.4 Full Wave Rectifier

### 6.2.2.1 Advantages of Full Wave Rectifier

- Efficiency of full wave rectifier is 81.2%. which is almost twice than the half wave rectifier.
- Ripple factor for full wave rectifier is 0.48
- Power is delivered during full cycle.

### 6.2.2.2 Disadvantages of Full Wave Rectifier

- Since in FBR two diodes conduct in series so the voltage drop in the internal resistance will be twice as HWR and this is objectionable when secondary voltage is small.
- Peak inverse voltage for FWR is  $2 \cdot V_{out}$ .

## 6.3 Rectifier Selection For our Design of LLC Converter

We are using full wave rectifier since it has low ripple factor and high efficiency. Power is delivered during full cycle of operation. HWR is rarely used since high ripple factor is produced and power is delivered only during half of the cycle. Its advantage is only its cheap ,simple and easy to construct

## 7 Efficiency

LLC Resonant converter has high efficiency because zero voltage switching is implemented.

Soft switching provides following benefits:

- High efficiency
- Reduction in circuit size because of the use of high frequency.
- Low EMI emissions owing to the absence of the higher order harmonics in the primary side current of the transformer .An EMI filter with high value of Q can be designed because of the inherent nature of the circuit, this also contributes to the low EMI emission.

However the efficiency decreases as the output power decreases with the changing load.

### 7.1 Efficiency At Resonance

At resonance, LLC converter has the maximum efficiency at the nominal output power. However as the power changes the efficiency changes.

### 7.2 Efficiency Below Resonance

Efficiency below resonance is lower than the other efficiencies since it has higher input current and higher conduction losses.

### 7.3 Efficiency Above Resonance

Efficiency above resonance is lower than the efficiency at resonance, since it has also higher input currents than that of resonance case and higher conduction losses. However it is higher than the efficiency below resonance.

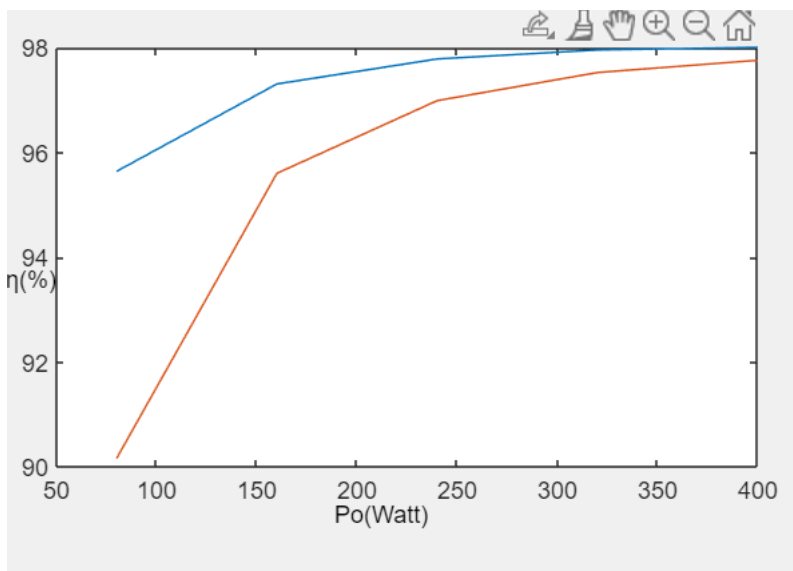


Figure 7.1 Efficiency at resonance (blue) Vs efficiency below resonance (orange)

## 8 Conclusion

LLC resonant converter has very high efficiency nearly 98% at resonance at full load which decreases slowly with changing the load. In over design it decreases to around 95% at 50% load. However efficiency below resonance and above resonance are slightly lower.

The main benefit of resonant converter is a soft switching on the primary and the secondary side as well. If we compare it to a soft switching topology like phase shift full bridge which is also widely used in SMPS applications especially for a higher power because it has the inherent soft switching capability and almost required in every application where is the high efficiency requirement.

If LLC is compared with phase shifted full bridge. The LLC is a full resonant converter and it achieves a soft switching on the primary and secondary side devices independent of the load. In terms of cost, the LLC could be implemented with a lower power components and also could be implemented with different circuits on the primary and secondary side like half bridge, full bridge and also a secondary could be different kind of rectifiers and that could cover a wide area of applications and power ranges.

The main drawback is challenging in the design and the controller scheme and protection and topology. It's a variable frequency Operation meaning that the voltage regulation of modulating the gain of that circuit is commanded by modulating frequency.

LLC resonant converter must Operate in the inductive region. In the capacitive region, the current is lagging the voltage meaning on the primary side the current on the body diode will have a hard commutation and that reverse recovery could cause a damaging or a failure condition on the primary MOSFETs and the another reason is in the inductive region current is leading the voltage and which cause zero voltage switching and that is the main benefit of the topology.

So in terms of normalized switching frequency it's less than one and higher one. At the resonance the switching frequency  $f_s$  equal to resonance frequency  $f_r$ . In other words if  $F_X$  is equal to one gain is equal to one and this is the best efficiency point to work. Therefore that's the point where



the nominal design is designed in terms of nominal input voltage and output voltage as well. But since the input voltage has a range that could go up and down beyond the nominal voltage in this case the tank has to operate in a boost gain or a buck gain. If input voltage is higher than the nominal voltage then boost gain operation is required and the FX will be left of the resonant point and this is the lower switching frequency. On the other hand if the required gain is less than one then it will be a buck gain and switching frequency has to be increased. Now, working below or above the resonant frequency or point, it's the different operation, different wave forms and different losses.

In the resonant operation switching frequency equal to the resonant frequency. In this case the resonant period is exactly equal to the switching period meaning that resonant sinusoidal waveform will finish it's resonant at the time where the switching frequency will also finish or ready to switch to second half. This is one half and in the second half it's also the same thing the current will resonate and completes for resonance in the negative direction. In this mode, the diode current or the rectifier current gets to zero at the point where the primary switch S1 is turned off.

Moving above the resonance operation  $f_s$  is larger than  $f_r$ , that means that the switching period is smaller. Meaning that the resonant waveform will not complete its sinusoidal cycle until interrupted by another switching period. The sinusoidal current doesn't get to zero or doesn't finish it's resonance before we starting a second half and similarly on the diode current  $I_{D1}$  and  $I_{D2}$  will start at 0 but they don't end at 0.

Now, below the resonance operation  $f_s$  is less than  $f_r$ , Therefore, switching period is much longer than that of resonance operation. In that mode the diode current still have a soft switching operation because the diode stop conducting before the new switching period starts. But the primary side does have because of that discontinuous operation or due to the extra time There will be extra circulating current and extra conduction losses in that mode. Each of these modes have different equations and behaviors in terms power loss and voltage regulation.

Transformer design is also a very critical step of LLC converter design, considering the skin effect due to the high frequency and the core losses. Copper ( $I^2 \cdot R$ ) losses are maximum in the case of

operation below resonance when the RMS value of the current is higher but also the skin effect losses will be higher at the higher frequency which is above resonance. For a good design to avoid the skin losses the conductor specifications for the transformer winding is chosen in such a way that the diameter of the conductor should be lower or equal to the skin depth at higher frequency to minimize the skin effect losses.

In the LLC one benefit is that tank could be implemented to be excited by a primary bridge to be a half bridge or a full bridge and each has its own positive and negative impacts. And each application is different, the half bridge has only two switches or only two MOSFETs and that's favorable from a cost point of view in some cases the input current is very high or the power level are very high. The full bridge could deliver the power with less RMS current in the primary side and that leads to a lower conduction loss. In HB for quite narrow voltage regulation quite wide frequency range is needed.

On the secondary rectifier the two options are commonly full Wave Rectifier(FWR) or a half Wave Rectifier(HWR). A FWR requires four devices and the HWR has only two devices but those devices voltage rating has to be twice the output voltage. Since in FWR two diodes conduct in series so the voltage drop in the internal resistance will be twice as HWR and this is objectionable when secondary voltage is small But HWR is rarely used since high ripple factor is produced and power is delivered only during half of the cycle. Its advantage is only its cheap ,simple and easy to construct.

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