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EXECUTIVE SUMMARY OF THE THESIS

## Gamer's Dilemma: An Evolutionary Game Theoretical Analysis

LAUREA MAGISTRALE IN MATHEMATICAL ENGINEERING - INGEGNERIA MATEMATICA

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### 1. Introduction and Research Objective

The Gamer's Dilemma is a problem in the ethics of virtual environments concerning an apparent asymmetry in the moral evaluation of fictional wrongdoing. Virtual murder is widely present in videogames and is usually regarded as morally permissible, while virtual child molestation is commonly judged to be morally unacceptable even when no real child is involved. Luck's original formulation shows the difficulty of sustaining this contrast: if both actions occur entirely within a virtual environment and neither directly harms an actual victim, it is not obvious which morally relevant distinction could justify accepting one while rejecting the other [3].

Most of the philosophical debate has therefore focused on the search for such a distinction, or on the attempt to weaken the intuitions that make the dilemma compelling. This thesis adopts a different perspective. Its aim is not to decide which moral evaluation of virtual actions is ultimately correct. Instead, it investigates how different evaluations may emerge, stabilize, and persist through the strategic interaction of the actors involved in the gaming ecosystem. In this sense, the thesis shifts the focus from the justification of a single moral judgment to the dynamics through which moral judgments be-

come socially established.

The starting point of the analysis is Arioli's static game-theoretic formulation of the Gamer's Dilemma [1]. In that framework, Developers, Society, and, in the extended version, Gamers are represented as strategic agents whose incentives determine which forms of virtual content can be sustained as equilibrium outcomes. The present work develops this approach in evolutionary terms. Static Nash equilibria are not treated as final answers, but as possible long-run configurations whose emergence depends on the adjustment process followed by the populations. The central question becomes: under which conditions do alternative normative regimes arise, and how are they selected over time?

To answer this question, the thesis constructs a sequence of evolutionary models based on multi-population replicator dynamics. Developers represent the producers of videogame content; Society represents the broader social environment that tolerates, or censors virtual actions; Gamers, in the three-population extension, represent consumers and their attitudes toward controversial content. The analysis combines local stability, invariant reductions, parametric investigation, and numerical simulations. Across these models, a common pattern emerges: the dynamics does not generally select a unique long-run outcome. Instead, it admits multi-

ple stable regimes, threshold effects, and path-dependent trajectories.

## 2. Mathematical Framework

The mathematical framework is evolutionary game theory. In a classical static game, strategic behavior is usually studied by identifying profiles from which no player has an incentive to deviate. Evolutionary game theory adds a dynamic interpretation: strategies that perform better than the population average increase their frequency, while strategies that perform worse decline. This makes it possible to study not only which equilibria exist, but also whether they can be reached, whether they are stable, and how the state space is divided among competing attractors [2, 4].

For a population with strategy distribution  $x = (x_1, \dots, x_n)$  and payoff functions  $\pi_i(x)$ , the replicator equation is

$$\begin{aligned}\dot{x}_i &= x_i(\pi_i(x) - \bar{\pi}(x)), \\ \bar{\pi}(x) &= \sum_{j=1}^n x_j \pi_j(x).\end{aligned}\tag{1}$$

The same principle extends naturally to multi-population systems. If population  $k$  has state  $x^{(k)}$  and payoff  $\pi_i^{(k)}$ , then

$$\dot{x}_i^{(k)} = x_i^{(k)}(\pi_i^{(k)} - \bar{\pi}^{(k)}).\tag{2}$$

This formulation is especially appropriate for the present problem, because the relevant actors do not belong to a single population. Developers, Society, and Gamers play different roles and adapt according to different payoff functions. Several structural properties of replicator dynamics are crucial for the thesis. First, the simplex of population states is invariant: strategy frequencies remain non-negative and sum to one. Secondly, every face of the simplex is invariant. Hence, if a strategy is absent at the initial time, it remains absent along the trajectory. This property justifies the analysis of lower-dimensional invariant subsystems. Thirdly, local stability can be studied through the Jacobian matrix of the reduced system. Pure equilibria with all transversal eigenvalues negative are locally attracting, while equilibria with eigenvalues of opposite sign act as saddles. These saddle

configurations are central because they may separate basins of attraction even though they are not observable as long-run outcomes.

The conceptual relevance of this framework lies in its ability to model moral and social patterns as dynamic outcomes. Stable equilibria can be interpreted as self-reinforcing normative regimes: once Developers, Society, and Gamers adopt mutually consistent behaviors, deviations become disadvantageous and the configuration may persist. Conversely, unstable equilibria and threshold manifolds explain how small differences in initial conditions can lead to different social outcomes.

## 3. Two-Population Evolutionary Model

The first model studies the interaction between Developers and Society. Developers choose among four strategies: including only murder-related content (*IM*), including only pedophilia-related content (*IP*), including both (*IB*), or excluding both (*EB*). Society chooses among tolerating only murder (*TM*), tolerating only pedophilia (*TP*), tolerating both (*TB*), or censoring both (*CB*). The numerical payoff matrices are inherited from Arioli's static formulation and are interpreted as an illustrative specification of the relevant incentives, not as empirical measurements.

The evolutionary analysis identifies two pure equilibria that are locally asymptotically stable. The first is (*IM*, *TM*), corresponding to a regime in which murder-related content is implemented by Developers and tolerated by Society. The second is (*EB*, *CB*), corresponding to a regime in which both forms of controversial content are excluded and censored. The eigenvalues of the reduced Jacobian are strictly negative at both profiles, confirming their local stability.

This result already reveals the first central conclusion of the thesis: the model does not determine a unique long-run convention. Both implementation-and-tolerance and exclusion-and-censorship can become self-reinforcing. Which of the two regimes is reached depends on the initial state of the populations and on the basin of attraction in which the trajectory begins.

To understand the mechanism of equilibrium selection, the thesis studies invariant reductions of

the full system. The most transparent one restricts Developers to *IM* and *EB*, and Society to *TM* and *CB*. Denoting by  $p$  the frequency of *IM* and by  $q$  the frequency of *TM*, the reduced dynamics is

$$\begin{aligned}\dot{p} &= p(1-p)(11q-3), \\ \dot{q} &= q(1-q)(3p-1).\end{aligned}\quad (3)$$

Besides the two stable boundary equilibria, this system admits an interior equilibrium

$$(p^*, q^*) = \left(\frac{1}{3}, \frac{3}{11}\right). \quad (4)$$

The Jacobian at this point has eigenvalues of opposite sign, so the equilibrium is a saddle. It does not represent a stable long-run state, but it acts as a threshold separating trajectories that converge to different attractors.

A richer reduction retains all four Society strategies while restricting Developers to *IM* and *EB*. This system contains two saddle configurations, denoted  $S_1$  and  $S_2$ , which extend the threshold mechanism beyond the planar case. Their saddle nature persists when they are embedded in the full six-dimensional system. Therefore, the reduced models do not merely describe artifacts of simplification; they reveal genuine organizing structures of the complete evolutionary game.

The analysis therefore shows that alternative moral configurations can persist within the same strategic environment. The relevant question is not only which equilibria are stable, but how the population trajectory is distributed among competing basins of attraction. Moral judgments, in this model, are not represented as isolated individual evaluations, but as social outcomes stabilized by reciprocal adaptation between content producers and the surrounding community.

## 4. Parametric Analysis

The numerical model shows that multistability and threshold effects can arise, but it does not by itself distinguish accidental features of the chosen numbers from structural features of the game. For this reason, the thesis introduces a parametric formulation of the two-population model. The Developer payoff includes the benefit of tolerated murder-related content  $m$ , the benefit of tolerated pedophilia-related content  $p$ , the costs of opposition  $k_M$

and  $k_P$ , and the reputational backlash  $b$  associated with pedophilia-related content. The parameters of society's payoff are  $\alpha_M$  and  $\alpha_P$  measure the social cost of tolerating murder-related and pedophilia-related content, respectively;  $\gamma_M$  and  $\gamma_P$  describe the cost associated with opposing those contents when they are present; and  $\delta_M, \delta_P$  represent the institutional or enforcement costs associated with censorship.

The first major result of the parametric analysis is the structural threshold

$$p = b. \quad (5)$$

When  $p < b$ , the reputational backlash associated with pedophilia-related content exceeds the benefit obtained when such content is tolerated. In this regime, *IP* is strictly dominated by *EB*, and *IB* is strictly dominated by *IM*. Hence the Developer's effective strategy set collapses from four strategies to two. This reduction is not imposed by moral assumption; it emerges endogenously from the incentive structure of the game. In the reduced regime  $p < b$ , the game can be further simplified. Depending on the sign of the parameter  $\delta_P$ , Society's effective strategy set is reduced to one of two equivalent pairs. A representative reduced interaction is  $(IM, EB) \times (TM, CB)$ . If  $x$  is the frequency of *IM* and  $y$  the frequency of *TM*, the system becomes

$$\dot{x} = x(1-x)((m+k_M)y-k_M), \quad (6)$$

$$\dot{y} = y(1-y)((\gamma_M-\alpha_M)x+\delta_M). \quad (7)$$

This two-dimensional system admits a complete qualitative classification. Depending on the signs of the threshold expressions, the dynamics may have a unique stable convention corresponding to implementation and tolerance, a unique stable convention corresponding to exclusion and opposition, a bistable coordination regime, or a cyclic regime organized around a non-hyperbolic interior equilibrium.

The bistable regime is particularly important. In that case both diagonal equilibria are locally stable and the interior equilibrium is a saddle. The saddle separates the two basins of attraction and determines which convention is selected. Thus the same strategic interaction can produce different long-run moral outcomes depending on the initial distribution of attitudes and behaviors.

The full regime  $p > b$  has a different structure. Pedophilia-related strategies are no longer eliminated by dominance, so all four Developer strategies remain active. The analysis of pure equilibria shows that only the four diagonal profiles can be Nash equilibria:

$$\begin{aligned} (IM, TM), & \quad (IP, TP), \\ (IB, TB), & \quad (EB, CB). \end{aligned} \quad (8)$$

Their local stability regions are determined by inequalities involving  $\delta_M$ ,  $\delta_P$ ,  $\alpha_M - \gamma_M$ , and  $\alpha_P - \gamma_P$ . In compact form, the stability conditions are

$$\begin{aligned} (IM, TM) : & \delta_P < 0, \quad \delta_M > \alpha_M - \gamma_M, \\ (IP, TP) : & \delta_M < 0, \quad \delta_P > \alpha_P - \gamma_P, \\ (IB, TB) : & \delta_M > \alpha_M - \gamma_M, \\ & \delta_P > \alpha_P - \gamma_P, \\ (EB, CB) : & \delta_M < 0, \quad \delta_P < 0. \end{aligned} \quad (9)$$

These regions show that the full game can support several different stable conventions. The long-run outcome depends on the parameter regime and on the basin in which the evolutionary process begins.

The lower-dimensional invariant faces provide the organizing structure of the full system. Among the  $2 \times 2$  faces, the dynamically relevant ones are those connecting pairs of diagonal equilibria. Under suitable conditions, they contain interior saddle equilibria that contribute to the separation of basins of attraction. The analysis of selected  $3 \times 3$  subsystems reveals an even more robust pattern. Introducing

$$\Delta_M = \alpha_M - \gamma_M, \quad \Delta_P = \alpha_P - \gamma_P, \quad (10)$$

all dynamically relevant  $3 \times 3$  subsystems exhibit the same qualitative classification: the interior equilibrium is a saddle when  $\Delta_M \Delta_P < 0$ , a center when both quantities are positive, and unstable when both are negative.

Finally, the thesis proves that the full  $4 \times 4$  system admits no fully interior equilibrium. Solving the Developer indifference conditions forces the component associated with the Society strategy  $TB$  to be zero. Therefore, coexistence of all strategies is impossible. The global dynamics must be organized by boundary equilibria, invariant faces, and saddle configurations. This reinforces the view that the evolutionary structure of the Gamer's Dilemma is not governed

by a single global coexistence point, but by a network of lower-dimensional threshold mechanisms.

## 5. Three-Population Extension

The final model introduces Gamers as a third population. Developers and Society retain their previous four strategies, while Gamers choose among accepting murder-related content ( $AM$ ), accepting pedophilia-related content ( $AP$ ), accepting both ( $AB$ ), or rejecting both ( $RB$ ). This extension captures the fact that the moral and strategic status of virtual content is not shaped only by producers and social institutions, but also by the attitudes of players themselves. The three-population system evolves on the product of three simplices and is again governed by multi-population replicator dynamics. The analysis identifies a significant structural degeneracy in the Society population. The payoff functions satisfy

$$\pi_{TM}^S = \pi_{CB}^S, \quad \pi_{TP}^S = \pi_{TB}^S. \quad (11)$$

As a consequence, the ratios  $y_1/y_4$  and  $y_2/y_3$  remain constant along trajectories whenever they are defined. Introducing the aggregate variable

$$u = y_1 + y_4, \quad (12)$$

one obtains

$$\dot{u} = 8u(1 - u)(x_2 + x_3). \quad (13)$$

Thus  $u$  is nondecreasing along trajectories and increases whenever Developers assign positive mass to the strategies involving pedophilia-related content. This property reduces part of the Society dynamics to a monotone process and reveals invariant quantities that organize the full system.

The static three-player game admits eight pure Nash equilibria. Under replicator dynamics, all of them are spectrally stable in the sense that all non-zero eigenvalues of the Jacobian are strictly negative. However, each equilibrium is non-hyperbolic because the Jacobian also contains zero eigenvalues generated by the structural degeneracies of Society's payoff functions. Linearization alone is therefore not sufficient to establish asymptotic stability in the full state space. The dynamics must instead be understood through invariant subsystems.

The most informative subsystem is obtained by imposing

$$x_2 = x_3 = 0, \quad (14)$$

so that Developers use only *IM* and *EB*. On this invariant set, all Society strategies receive the same payoff, and therefore Society becomes frozen at its initial condition. Its state nevertheless affects the subsequent evolution through the parameter

$$a = y_1 + y_3. \quad (15)$$

After eliminating dominated Gamer strategies, the reduced interaction involves only *IM* and *EB* for Developers and *AM* and *RB* for Gamers. If  $p$  denotes the frequency of *IM* and  $r$  the frequency of *AM*, the planar system is

$$\dot{p} = p(1 - p)((8a + 4)r + 5a - 9), \quad (16)$$

$$\dot{r} = 5apr(1 - r). \quad (17)$$

A qualitative transition occurs at

$$a = \frac{5}{13}. \quad (18)$$

For  $a < 5/13$ , the frequency of *IM* decreases throughout the interior and the dynamics is driven toward *EB*. For  $a > 5/13$ , the nullcline enters the state space and creates a region from which *IM* can become dominant together with increasing acceptance by Gamers. At the threshold itself, the system lies at the boundary between these regimes.

This result is important because the parameter  $a$  is determined by the initial composition of Society. Even when Society is dynamically frozen in the reduced subsystem, its initial distribution shapes the long-run interaction between Developers and Gamers. The three-population extension therefore confirms the central insight of the thesis: equilibrium selection is path-dependent and mediated by invariant structures and threshold effects.

## 6. Conclusions

The main contribution of the thesis is the introduction of an evolutionary perspective on the Gamer's Dilemma. Instead of treating the dilemma only as a question about the correct moral evaluation of virtual murder and virtual

child molestation, the thesis studies how different moral evaluations can arise as stable outcomes of strategic interaction.

The two-population numerical model shows that multiple self-reinforcing regimes can coexist. The parametric analysis demonstrates that this phenomenon is not accidental, but follows from structural properties of the game. The three-population extension confirms that the inclusion of Gamers enriches the strategic landscape without eliminating multistability, threshold dependence, or the role of invariant subsystems.

Across all chapters, stable equilibria represent possible normative conventions, while saddle equilibria and lower-dimensional invariant faces determine how those conventions are selected. The long-run outcome depends not only on payoffs, but also on initial conditions and on the historical trajectory of the system. This provides a formal account of how alternative moral judgments may become socially established over time.

The philosophical implication is that disagreement about virtual actions need not be interpreted only as a failure to identify the correct moral principle. It may also reflect the existence of multiple stable normative equilibria. From this perspective, the Gamer's Dilemma is not only a problem about whether a morally relevant distinction exists, but also a problem about how moral distinctions emerge, persist, and acquire social force.

The value of the analysis does not lie in providing an exhaustive classification of every possible trajectory, but in identifying the structural mechanisms that organize the evolutionary behavior of the model. The numerical, parametric, and three-population analyses all point to the same conclusion: moral judgments in virtual environments may be shaped by stable conventions, threshold effects, and path-dependent processes. In this sense, the thesis shows that evolutionary game theory provides a productive framework for studying the formation of moral judgments and for connecting ethical debates with formal models of social dynamics.

## References

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