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Applications of centrality Indices to assess performance reduction risk in transportation networks due to disruption or failure.

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Abstract

Transportation networks are the backbone of modern civilization in our linked globe, supporting the movement of people, products, and services. Disruptions and performance degradation threats in these networks, on the other hand, might have far-reaching implications. This thesis investigates the use of centrality indices, a set of network analytic tools, to analyse and manage the risks of performance degradation in transportation.

Using real-world data, researchers use centrality indices to identify critical nodes and linkages within transportation networks. This method aids in identifying potential network security threats, where attacks on these critical elements may result in performance degradation. The study delves into the complex landscape of threats affecting transportation performance, assessing the impact of variables such as traffic congestion, accidents, and natural disasters."

The study's findings highlight the importance of centrality indices in risk assessment and management within transportation networks. They can, in fact, be used to improve network resilience by identifying critical nodes and linkages, implementing preventative measures, and developing effective contingency plans.

Understanding and managing performance decrease risks is critical in a world where transportation networks play a critical role in the global economy. This thesis adds to the body of knowledge by illustrating how centrality indices may be used as effective tools for risk assessment and decision-making in the transportation domain.

Keywords: Centrality Indices, Risk Assessment, Disruption, Failure, Resilience, Transportation Networks

Abstract in lingua italiana

Le reti di trasporto sono la spina dorsale della civiltà moderna nel nostro mondo interconnesso e supportano il movimento di persone, prodotti e servizi. Le interruzioni e le minacce di degrado delle prestazioni in queste reti, d'altra parte, potrebbero avere implicazioni di vasta portata. Questa tesi studia l'uso degli indici di centralità, un insieme di strumenti analitici di rete, per analizzare e gestire i rischi di degrado delle prestazioni nei trasporti.

Utilizzando dati reali, i ricercatori usano gli indici di centralità per identificare i nodi e i collegamenti critici all'interno delle reti di trasporto. Questo metodo aiuta a identificare le potenziali minacce alla sicurezza della rete, dove gli attacchi a questi elementi critici possono provocare un degrado delle prestazioni. Lo studio si addentra nel complesso panorama delle minacce che influenzano le prestazioni dei trasporti, valutando l'impatto di variabili come la congestione del traffico, gli incidenti e i disastri naturali".

I risultati dello studio evidenziano l'importanza degli indici di centralità nella valutazione e nella gestione del rischio all'interno delle reti di trasporto. Possono, infatti, essere utilizzati per migliorare la resilienza della rete identificando i nodi e i collegamenti critici, implementando misure preventive e sviluppando piani di emergenza efficaci.

Comprendere e gestire i rischi di riduzione delle prestazioni è fondamentale in un mondo in cui le reti di trasporto svolgono un ruolo critico nell'economia globale. Questa tesi si aggiunge al corpus di conoscenze illustrando come gli indici di centralità possano essere utilizzati come strumenti efficaci per la valutazione del rischio e il processo decisionale nel settore dei trasporti.

Parole chiave: Indici di centralità, valutazione del rischio, interruzione, guasto, resilienza, reti di trasporto.

Contents

Introduction

The movement of commodities and people throughout the world is undergoing extraordinary changes. The need for effective transportation networks is greater than ever as urbanization and population growth pick up speed. The movement of people, products, and services is made easier by transportation networks, which constitute the lifeblood of contemporary society. This promotes social advancement and economic expansion. However, because transportation networks are dynamic and subject to a wide range of threats, creative solutions are needed to maintain their resilience.

Risks associated with performance degradation in transportation networks are complex and sometimes difficult to forecast and control. These risks have a significant influence on the dependability and functionality of transportation systems, ranging from traffic congestion in booming metropolises to supply chain interruptions caused by unanticipated events. Understanding and mitigating performance decrease threats is not just a responsibility, but an essential Scope in an era when our reliance on these networks has never been stronger. We now discuss the idea of centrality indices in this context. These are a complex yet effective collection of tools that come from network research. Due to its ability to identify important nodes and connections inside complex systems, centrality indices have attracted a great deal of attention from a variety of disciplines. Although these indicators are widely used in fields like statistics and social network analysis, there continues to be little study on how to use them to evaluate the risks of performance loss of transportation networks.

The primary goal of this thesis is to investigate the use of centrality indices as tools for identifying and managing performance decrease threats in transportation. Our key research question is: ***Can centrality indices give important insights into critical components of transportation networks, allowing us to proactively manage and avoid performance decrease risks?*** Our research intends to contribute to the current body of knowledge and fill a gap in the literature by putting light on the untapped potential of centrality indices around transportation. This thesis has important practical ramifications in addition to being an intellectual undertaking. By using centrality indices in transportation risk assessment, planners, stakeholders, and decision-makers can be better equipped to deploy resources wisely, make well-informed decisions, and strengthen the resilience of transportation networks. By utilizing the analytical power of centrality indices, transportation systems may be made more resilient and flexible in a world where the efficient and dependable flow of people and products is essential to economies and civilizations. The theory and use of centrality indices in the context of transportation networks will be examined in the next chapters, where we will also offer research findings that highlight the transformational potential of these indices in the quest of network performance and risk reduction.

1. Centrality Measures

1.1 Generalities on Centrality Measures

Centrality measurements are useful in network studies and graph theory because they highlight critical nodes within a network. “Since every centrality measure takes into consideration different features of networks, nodes can be central for an index, but less important for another one. Hence, it is important to deeply understand the ratio behind each index, and to compare outcomes of different indices to have a comprehensive understanding of the network”[1] They aid researchers in understanding and representing the significance of nodes in networks. These measurements are commonly used to identify prominent participants in big and complicated social networks.[2].

In the context of networks, centrality refers to measures that quantify the relative significance or impact of nodes within a network. Centrality measurements are important in network analysis because they help anticipate the properties and importance of nodes. There are several centrality measures to choose from, including degree centrality, betweenness centrality, and closeness centrality. These measurements evaluate several elements of a node's significance, such as connection, influence in information flow, and closeness to other nodes.[1]

Here are some key centrality measures and their contributions:

1. Degree centrality: A node's popularity or influence is determined by the number of connections it has.[3]
2. Closeness centrality: Indicates an efficient means of information dissemination by measuring the speed at which a node can connect to other nodes.[3]
3. Betweenness centrality: Indicates a node's control over information flow by measuring how far it is located on the shortest paths between other nodes.[3]

These centrality measurements aid in the identification of key nodes in a network by quantifying their attributes and significance. [1] By offering information on node locations, routes, walks, and geodesics, they improve our comprehension of network architecture. [4] When several centrality measurements are applied to real-world networks, the results may be comparable, highlighting the robustness of certain network structures. However, it is critical to recognize that divergent centrality indices, which are designed to capture different aspects of network dynamics, may produce incomparable results. This nuanced understanding emphasizes the significance of selecting appropriate centrality measures that are aligned with the network's specific characteristics and objectives. [5] All things considered, centrality measurements are essential to network analysis since they offer insightful data regarding the significance and role of nodes inside a network. [2]

1.2 The Classical Centrality Index

1.2.1 Degree Centrality Measures

The number of edges incident to a vertex determines its degree of importance, which is a basic concept in network theory. It is the most basic notion of centrality and has been applied extensively “to analyse networks of different kind”. Higher degree centrality denotes a more central or significant vertex in the network. Degree centrality is computed by counting the number of edges connected to a vertex.

Degree centrality is a key notion in network analysis, providing a basic measure of a vertex's importance in a network based on its connection.[6]

The following are some crucial details regarding degree centrality's significance in network analysis:

1. Fundamental concept: Degree centrality, which rates a Node’s significance in a network's structure and dynamics, is the historically first and conceptually most basic centrality concept.[6]

2. Widely used: One of the most used metrics for determining a node's significance within a network centrality.[3]

3. Connection and influence: It is Centrality gauges a node's degree of influence and connection by counting the edges that incident on a vertex.[6]

In general, degree centrality is an essential metric in network analysis since it aids in the identification of significant nodes and the comprehension of the dynamics and structure of networks.[3] [6]

A mathematical model that uses a node's number of edges to determine how important it is in a network. The fractional information centrality shows shifts in centrality ranks as the fractional parameter fluctuates, and it may be computed using the graph Laplacian and its fractional analog. The shift of the fractional information centrality from a local to a global centrality measure is revealed by the link between the fractional information centrality and the degree centrality.[6][7]

The calculation of degree centrality is shown in Equation.

$$C_D(j) = \sum_{j=1}^{n} A_{ij}$$

$C_D(j)$represents the degree centrality of node (j).

A denotes the adjacency matrix of the graph.

The formula for degree centrality in a symmetric matrix and the formula for calculating out-degree centrality for an asymmetric matrix are the same. On the other hand, the usual formula for determining in-degree centrality is inverted, adding down the columns j instead of across the columns i equation.

It is important to note that in an undirected network, the node for which you are calculating the centrality cannot be connected to itself in a simple graph; therefore, "Number of connections to the node" represents the number of edges or links connected to the node you are interested in. "Total number of nodes - 1" is used to normalize the Degree Centrality. It is subtracted by one to exclude the node you are calculating the centrality for from the denominator. In a directed network, the formula changes slightly:

In-Degree Centrality (C_in): C_in = (Number of incoming connections to the node) / (Total number of nodes - 1)

Out-Degree Centrality (C_out): C_out = (Number of outgoing connections from the node) / (Total number of nodes-1).

Well, for directed networks, you must sum of rows to get the out-degree and on columns to get the in-degree. For undirected networks, rows and columns give the same result because the adjacency matrix is symmetric.

Higher degree centrality values indicate nodes that are more central or well-connected within the network. Degree Centrality values vary from [0, n-1]. where n is the number of nodes of the network. Furthermore, degree 0 means that the node is connected to no other node of the network, and so the network itself is not connected. This does not happen in transportation networks.

The network's adjacency matrix-based methods can be used to efficiently do this computation. [8]

It is a measure for interpreting a node's significance or centrality inside a network.

1. Because degree centrality numbers are based on the number of direct connections a node has, they might be affected by the local feature of a network.[7]

2. Other centrality measures, such as information centrality, should be considered. Information centrality expresses a node's global significance within a network by accounting for not only its direct connections but also the indirect influence it has across the entire network structure.[7]

In network analysis, comparing nodes according to their degree centrality is standard procedure.

Many centrality metrics and techniques for determining prominent nodes according to their degree of centrality are covered in several abstracts. These are the main ideas:

Edge Centrality is taken into consideration: Presents the idea of nearest-neighbor edge centrality, which gauges the significance of edges within a network. Although degree centrality is a commonly used metric for vertices, an edge centrality idea comparable to vertex degree does not yet exist. Nonetheless, a notion known as nearest-neighbor edge centrality has been put forth that can recognize central edges in both real-world networks and network models.[6]

The concept of nearest-neighbor edge centrality is introduced, which takes edge centrality into account. This metric measures the importance of relationships between adjacent nodes to assess the significance of edges within a network. The centrality of nearest-neighbor edges reveals information about the local connectivity and influence of edges.

Limitations:

It is important to keep in mind that degree centrality has certain restrictions and disadvantages.

1. Unimodular behaviour: A study revealed that very large and very small groups are not very central, suggesting that a group's centrality value rises with size but eventually begins to fall. [9]This implies that the significance of very tiny or large groups in a network may not be fully captured by degree centrality.

2. Limited information regarding node positions: The number of edges incident to a node determines degree centrality, which is a basic indicator of significance but could miss other crucial details of a node's placement within the network.[4]. In some

circumstances, additional centrality measures like betweenness centrality and closeness centrality may offer a more thorough knowledge of node importance. [10]

3. Suitability in psychological networks: Because of presumptions that might not hold true in this situation, degree centrality, betweenness centrality, and closeness centrality may not be appropriate indicators of node importance in psychological networks. More suitable centrality measurements might be ones that consider the unique traits of psychological networks.[10].

In conclusion, degree centrality is a commonly used measure, but it has limits when it comes to accurately representing the significance of nodes in a network, especially when it comes to very small or huge groups as well as certain network types like psychological networks. In these situations, a more thorough understanding of node importance might be possible using other centrality methods. In some cases, degree centrality might not give an accurate representation. Here are a few justifications:

1. Fractional centralities: The fractional degree centrality, which is primarily driven by the local aspect, is unable to capture changes in node centrality rankings that the fractional version of the information centrality, which considers both local and global aspects, can produce.[7] The fractional degree centrality quantifies the fraction of neighbors a node connects to within a network, which is primarily driven by the local aspect. It cannot, however, capture changes in node centrality rankings. The fractional version of information centrality, on the other hand, takes into account both local and global aspects, providing a more comprehensive measure of a node's significance by accounting for both direct and indirect influences throughout the network.

In summary, Degree centrality is a local centrality measure that counts the number of edges that intersect a node to determine its importance. This measure is relevant to complex networks and can be used to compute group centrality estimations. Instead of focusing on individual nodes, group centrality often includes analyzing the centrality of a group of nodes collectively.

1.2.2 Betweenness Centrality Measure

Betweenness centrality is a measure that quantifies a single vertex's tendency to be more central than all other vertices in a graph. [11] The number of shortest paths that pass through the vertex determines its percentage. From the perspective of network analysis, it’s crucial for locating important nodes and connections inside a network. A graph is said to be betweenness-uniform if its betweenness centrality is the same at every vertex. These graphs have intriguing characteristics, like the fact that they are either 3-connected graphs or K-regular graphs Furthermore, tiny diameters are typically found in betweenness uniform graphs with high maximal degree. [12]. The idea behind the mathematical definition of betweenness centrality is that a node can potentially affect the information traveling down a path if it is located on the shortest path between two other nodes.[13] Different centrality measures: In network analysis, several centrality measures are employed. However, betweenness centrality is particularly significant as a measure that captures a node's impact in linking others in the network. Compared to other centrality measures like degree centrality, it differs conceptually.

In terms of shortest pathways, it measures the frequency with which a node appears between pairs of other nodes. Many measures have been put out to quantify betweenness centrality, such as Random Walk Betweenness Centrality, Stress Centrality, and Betweenness Centrality. The classic formula for calculating betweenness centrality (B) for a network node i is as follows:

$$B_{(i)} = \Sigma_{s \neq i \neq t} \frac{\sigma_{st}(i)}{\sigma_{st}}$$

σ_{st} is the total number of shortest paths between nodes s and t,

$\sigma_{st(i)}$ is the total number of paths that pass-through node i.

Axiomatic methods have been used to examine and characterize these measurements.[14] Applications for it can be found in many domains, including as information dissemination, network control, and social network analysis.[15] [16] Find all the shortest pathways in the network between pairs of nodes, then count how many of those path's travel through each node to calculate betweenness centrality.[17]

Betweenness Centrality Equation (Brandes Algorithm):

$$B_{(v)} = \Sigma_{s \neq v \neq t} \frac{\sigma_{st}(v)}{\sigma_{st}}$$

It finds all the shortest paths in the network between two nodes, counts how many of those paths pass through each node, and calculates betweenness centrality.

$B_{(v)}$ is Betweenness centrality of node v .σ_{st} Total number of shortest paths from node s to node t . $\sigma_{st(v)}$ Number of those paths that pass-through node v.

The equation is derived from the Brandes algorithm, which is widely used for computing betweenness centrality in complex networks. In particular, Maccari's research has helped to improve the efficiency and scalability of algorithms for calculating betweenness centrality in large-scale networks.

The Brandes algorithm improves the computation of betweenness centrality by efficiently counting the number of shortest paths and those that pass through each node. Maccari's contributions could include additional optimizations or performance improvements to the algorithm, allowing it to be applied to larger and more complex networks.

To calculate betweenness centrality, several techniques are available, including the O(n^3) tipass-through randes algorithm.[18]. Additional techniques, such as the one put out by Maccari et al., are designed to increase the computational efficiency of betweenness centrality in distributed networks.[19]

The following are the ways that betweenness centrality varies for directed and undirected networks:

1.Undirected Networks: The average shortest distance between nodes in connected components determines the betweenness centrality in undirected networks. The relationship between average shortest distances and betweenness in undirected networks is disclosed by the betweenness identities.[14]

2. Directed Networks: According to their function, such as a transition node inside a cluster or across clusters, this model characterizes nodes' behaviour. In directed networks, it offers a node classification and strengthens dense communities. A novel centrality model based on random paths betweenness centrality is applied in directed networks. [20]

In network analysis, this is a crucial node influence measure. By counting the number of shortest paths that travel through a node, it evaluates a node's potential influence on a network. The following are some essential details about betweenness centrality's significance:

1. Node influence: The degree to which a node serves as a link or intermediary between other nodes in a network is measured by betweenness centrality. Nodes with high betweenness centrality can regulate the movement of resources or information throughout the network.[13]

Several effective techniques have been put forward to calculate betweenness centrality.

k-step BC and k-step GBC: These betweenness centrality variations calculate the probability that information will go along the shortest paths in a predetermined amount of transmission time.[15]

Betweenness Centrality k-step (BC k-step):

1. The chance that information will travel down the shortest paths inside a network within a certain length of transmission time (k steps) is calculated by k-step BC.

2. Rather of considering all feasible paths, k-step BC concentrates on the shortest paths that can be taken within a given time period or transmission budget (k steps). It takes into account the amount of time it takes for information to transit between nodes.

k-step Group Betweenness Centrality (k-step GBC):

1. K-step GBC, like k-step BC, extends the concept to group dynamics. It computes the likelihood that information will flow down the shortest paths among a set of nodes within a given transmission time (k steps).
2. In the context of k-step GBC, the analysis is focused on groups of nodes as well as individual nodes. The metric sheds light on how information can travel efficiently within a group in a short amount of time.

Both types have a temporal component by taking into account the number of steps or time units necessary for information transfer. These measurements are especially relevant in cases where the amount of time it takes for information to transit between nodes is essential.

2. Approximation algorithms: To expedite betweenness centrality computations, several approximation algorithms have been created. The computing time can be drastically decreased by using these algorithms. Approximation algorithms for betweenness centrality use a variety of techniques to produce faster computations, such as randomization, single-pass tactics, parallel processing, and heuristics. These techniques are critical in dealing with the computing challenges of centrality analysis in big and complicated networks. The specific choice of an approximation method is determined by the network's features and the desired balance of speed and accuracy.

4. Distributed algorithm: With just minor adjustments to distance-vector routing protocols, an effective algorithm has been developed for calculating betweenness centrality in a distributed Network [19]

5. borders computation: To decrease the number of candidate groups of nodes and increase efficiency, a method has been developed to compute boundaries on the group betweenness centrality of groups of vertices.[21]

6. Incremental approach: Rather than starting from scratch, an incremental technique has been designed to compute betweenness centrality in dynamic networks.

The incremental technique to computing betweenness centrality in dynamic networks is a strategy that efficiently updates centrality scores as the network changes over time. Instead of recalculating centrality from scratch each time the network topology changes, the incremental technique updates the existing centrality values to integrate the changes more efficiently. Dynamic networks are those that alter over time. Changes can include the addition or removal of nodes, the addition or deletion of edges, or changes to the weights of existing edges.

The following equation the betweenness centrality of a node v:

$$g(v) = \sum_{s \neq v \neq t} \frac{\sigma_{St}(v)}{\sigma_{St}}$$

Where σ_{St} total number of shortest paths from node S to node t and $\sigma_{St}(v)$ is the number of those paths that come through v (where v is not an end point.

According to the summation indices, a node's betweenness centrality grows with the number of pairings of nodes. As a result, the computation may be rescaled by dividing through by the number of node pairings that do not include v.

The measure of betweenness centrality, indicated by g(v), is not naturally constrained to the range [0,1]. However, for the purposes of normalization, it is usual in directed graphs to divide by (N-1)(N-2)/2, and in undirected graphs to divide by (N-1)(N-2)/2. This normalization is used to scale values inside a specified range and make comparisons across networks easier, where N is the number of nodes. It should be noted that this scales for the greatest possible value when every single shortest path crosses one node. This is frequently not the case, and normalization can be conducted without sacrificing precision.

Normal $(g(v)) = \frac{g(v)-min(g)}{max(g)-min(g)}$ Where the results in max(normal) = 1 ;Min(normal) = 0 where min(g) represents the minimum betweenness centrality value in the network, and max(g) represents the maximum. The normalized values obtained, given as Normal $(g(v))$, range from 0 to 1. It is important to note that this normalization method is network-specific and may not be appropriate when comparing betweenness centrality values across different classes of networks.

All things considered, these algorithms offer effective methods for calculating betweenness centrality in various network settings, enhancing computation speed and accuracy.

It is important to note that this is always a scaling from a smaller range into a wider range, thus no accuracy is lost.

Higher values denote greater centrality. When the index is normalized, the betweenness centrality values normally vary from 0 to 1. Because it is located on numerous shortest paths that connect other Nodes, a vertex with a high betweenness centrality value is thought to be more central and influential in the network.[22][13].

Understanding the significance of a vertex in a graph can be gained by interpreting high and low betweenness centrality values.

1. High betweenness centrality: When a vertex in a graph has a high betweenness centrality score, it means that it is important for connecting other vertices. It implies that the vertex may be a bottleneck or bridge in the network since it is located on numerous short paths connecting other vertices.

2. Low betweenness centrality: Conversely, a vertex with a low betweenness centrality score may have less of an impact on the connections it makes with other vertices. It can suggest that the vertex is not a vital bridge or connectivity in the network.

When evaluating betweenness centrality ratings, it's critical to take the specific context and goal of the network study into account.

In conclusion, vertices with high betweenness centrality scores are critical for forming short connections with other vertices, whilst those with low scores are thought to have less influence. Nonetheless, the interpretation needs to take the particulars of the network under research into account and be dependent on the surroundings. [4][10][22]

Evaluating applications of betweenness centrality in transportation and infrastructure planning.

1. Identifying critical sites in road networks necessitates consideration of both topological and geographical factors. A modified type of betweenness centrality known as origin-destination betweenness centrality was used in this scenario. To identify crucial spots in a transportation network, this modified measure considers both topological traits and geographical considerations.

2. Determining the number of passengers drawn to public transportation stations: It was discovered that network centrality analysis, which includes betweenness centrality, was a helpful indication for determining the number of passengers at public transportation stations. [23]

These results suggest that betweenness centrality can be utilized in infrastructure and transportation planning to evaluate the configuration of road networks, pinpoint important sites, and calculate the number of passengers drawn to public transportation hubs. [24][25][23]

There are various restrictions and difficulties related to betweenness centrality that need to be taken into account:

Shortest path assumption: Betweenness centrality assumes that data in the network travels along the shortest paths. In real-world networks, where information may travel via several paths, this could not always be the case.

Computational complexity: It can be computationally costly to calculate betweenness centrality for big networks. To expedite these computations, three approximation techniques have been created.

Graph operations: A composite graph's nodes' betweenness centrality may vary depending on its structure. When a composite graph is created by joining various graphs, the betweenness centrality of nodes within the composite graph can vary. This variation is due to the structural properties established during the process of joining different graphs, which alter the flow of information and the relative importance of nodes in the final composite network.

amalgamation and merging are graph procedures that entail joining nodes and edges to produce composite networks. These procedures can be used to investigate the betweenness centrality of composite networks and to comprehend how the combining of different topologies affects the flow of information or influence within the final network.[13][26][27]

In Following cases, betweenness centrality might not be the best measure.

1. The assumption that every connected betweenness-uniform graph is either a cycle or a three-connected graph means that the uniformity in betweenness centrality is specifically seen in these two categories. When a graph does not fall into the cycle or three-connected categories, it implies that betweenness centrality may not exhibit the same uniform patterns, allowing for more sophisticated graph topologies that differ from this observed uniformity.[12]

3. Spatial arrangement of network elements: Betweenness centrality computations may be biased due to the spatial density of nodes. A factor to consider in network analysis where the spatial density of nodes might induce bias in estimates of betweenness centrality. This bias occurs when densely inhabited areas may have greater centrality values mistakenly, thus influencing the understanding of the network's structural relevance.

Betweenness centrality offers an alternative viewpoint on node relevance when compared to other centrality measures like degree centrality, whereas degree centrality counts the number of connections a node has. Conversely, betweenness centrality emphasizes a node's function in tying other nodes in the network together. Although each of the two centrality measures has advantages and disadvantages of their own, betweenness centrality is especially helpful in locating nodes in the network that serve as bottlenecks or bridges. [12][11][10]

The Benefits

1. Measures node importance: Betweenness centrality calculates a node's relevance based on its position in the network.

2. Identifies central nodes: In a network, nodes with high betweenness centrality are frequently regarded as its center nodes. [4]

Negative aspects:

1. Computational complexity: It can be computationally costly to calculate betweenness centrality for big networks.

Complementarity:

Betweenness centrality is a measure that evaluates a node's importance in linking other nodes in a network. Here are some important conclusions and revelations regarding betweenness centrality:

1. Axiomatic analysis: One of the medial centrality metrics that evaluates a node's contribution to linking other nodes in the network is betweenness centrality. It is predicated on the idea that data in networks takes the shortest paths. On the other hand, Random Walk Betweenness Centrality assumes that data moves at random along the edges.

2. characteristics of graphs that are betweenness-uniform: Three-connected graphs or cycles are examples of betweenness-uniform graphs. Furthermore, tiny diameters are found in betweenness uniform graphs with high maximal degree.[12]

3. Connection to additional centrality metrics: The foundation of betweenness centrality is the distinction between the most central vertex's centrality and all other vertex's centralities.[13][22]

1.2.3 Closeness Centrality Measure

Closeness Centrality is a distance-based centrality measure that assigns a node's rank in a graph according to how close it is to other nodes. Closeness centrality is a useful measure for assessing node relevance in a variety of contexts, including spatial network facility placement issues. [28] The information content and flow in networks can be described using information elements and partial ordering. [29]

Cross-Layer Closeness Centrality (CCC): A novel measure known as cross-layer closeness centrality (CCC) has been defined in the context of multiplex social networks or multi-layer networks to calculate a node's degree of closeness to each other node in the network. [30] The CCC is calculated by considering the shortest paths that span different layers of the multiplex network. The goal is to capture both intra-layer and inter-layer connections, providing a more comprehensive measure of a node's centrality across the entire multiplex structure.

While classical closeness centrality focuses on the shortest paths inside a single layer, CCC applies this notion to multiplex networks by combining data from many layers. It simply combines intra-layer closeness centrality with inter-layer connections, providing a more nuanced view of a node's proximity within the larger multiplex social network. CCC presents a valuable technique for capturing the deep relationships inside multiplex networks, providing insights regarding node centrality that extend beyond the bounds of standard closeness centrality.

The closeness centrality of vertices in a variety of graph forms, including the shadow graph, complementary prism, edge corona, and disjunction of graphs, can be calculated using derived formulas. [31]

$$C_B(x) = \frac{1}{\Sigma_y \, d(y,x)}$$

Where $d(y,x)$ is the distance (shortest route length) between vertices x and y [32]People commonly refer to closeness centrality in its normalized version, which indicates the average length of the shortest pathways rather than their total. It is commonly provided by multiplying the preceding formula by N-1, where N is the number of nodes in the graph, resulting in:

$$C(x) = \frac{N-1}{\Sigma_y \, d(y,x)}$$

The normalizing of closeness makes it easier to compare nodes in graphs of varying sizes. For big graphs, the negative one in the normalisation becomes insignificant and is frequently deleted.

Closeness centrality can be computed by means of a modifies breadth-first search algorithm as follows. Starting with a node, the algorithm finds its neighbors, then the neighbors, and so on, until every node has been examined. Each visited node's distance from the starting node is measured, and the reciprocal of the total of these distances is used to compute its closeness centrality.[33]

The following equation describes the described algorithm for computing closeness centrality via a modified breadth-first search:

$$C_C(v) = \frac{1}{\sum_{u \neq v} d(v,u)}$$

Where,

$C_C(v)$ is the closeness centrality of node v

$d(v,u)$ represents the shortest path distance from node v to node u

The sum is taken over all nodes u that are not equal to v measuring the reciprocal of the total of these distances.

It measures the average separation between a certain node and every other node in the network.[30][34]

Closeness centrality can also be calculated using shortest-path graphs. These trees can be used to determine the inverse relationship between the logarithm of degree and closeness centrality, as well as to estimate the network topology. In this context, "estimating network topology" refers to the use of certain structures, most commonly trees, to study and infer the network's overall organization and connectivity patterns. A special component of this analysis is the inverse relationship between the logarithm of degree and closeness centrality, which provides insights into the interaction between node degrees and their network centrality.

Closeness centrality can also be calculated using shortest-path graphs. These trees can be used to determine the inverse relationship between the logarithm of degree and closeness centrality, as well as to estimate the network topology.[35]

The following instances demonstrate how closeness centrality for nodes in a network is calculated:

1. Distributed Detection: Each node in a distributed network uses a breadth-first search technique to approximate its own closeness centrality, which it then shares with other nodes.[33]

Shortest-Path Tree: A tree structure known as a shortest-path tree covers all the nodes in a network, with each edge denoting the shortest path between a root node and all other nodes. It is a depiction of the shortest routes between the network's root and every other node. The "inverse of closeness centrality" is the reciprocal of closeness centrality. The degree to which a node is close to every other node in the network is measured by its closeness centrality. The inverse of closeness centrality within the shortest-path tree's structure is directly proportional to the degree logarithm of the node. This suggests that in the shortest-path tree, nodes with greater degrees typically have lower inverse closeness. Nodes with greater degrees in the tree may have a different closeness centrality profile than nodes with lower degrees, according to a pattern suggested by the link between the inverse of closeness and the logarithm of degree in the shortest-path tree.

2. Approximation of Closeness Centrality via Shortest-Path Tree: The inverse of closeness is directly proportional to the logarithm of degree in a shortest-path tree.[35]

When analyzing networks, closeness centrality is a crucial indicator of node influence. Calculating closeness centrality requires a lot of work and time, particularly when separating graphs. [36] Although there are different centrality measures and modifications to consider based on the environment and analytic aims, Closeness centrality is an effective measure for evaluating node accessibility and influence in networks.

The information content and flow in networks can be described using information elements and partial ordering. [29]

In complex network systems, the significance of nodes and edges can be ascertained by analyzing flow adjacency matrices and network context. A Flow adjacency matrix is a square matrix in which each element (i, j) denotes the degree or intensity of flow in the network from node i to node j. The flow can depict a variety of interactions, including resource exchange, influence, traffic, and information transfer. When working with directed networks, where interaction direction is important, this matrix is extremely helpful.[37]

The following insights can be used to interpret closeness centrality values, both high and low.

A node with a high closeness centrality score is close to many other nodes and has a rapid path to every other node in the network. A node with a low closeness centrality score may be difficult to reach in the network due to its relative distance from other nodes.[30]

It is crucial to remember that the network context—such as psychological networks—may affect how appropriate closeness centrality is as a gauge of node importance. Due to assumptions that could not be consistent with the interactions between psychological variables, betweenness and closeness centrality may be less appropriate as indicators of node importance in psychological networks.[10]

In conclusion, low closeness centrality values imply limited reachability, whereas high scores show that a node is well-connected and can swiftly reach other nodes. The specifics and presumptions of the network under analysis must be considered, as the interpretation may change based on the network context.[10][30]

Effective calculation: Developing methods that split large networks into biconnected components and employ incremental update approaches to maintain computation results when graphs change is one way to solve the time-consuming nature of accurate closeness centrality calculations.[38] Subgraphs that are maximally biconnected within a graph are called biconnected components. Stated differently, it is a connected subgraph that maintains its connectivity even if any vertex (node) or edge is eliminated. In graph theory, biconnected components are essential, especially when it comes to recognizing strong network architectures.

The drawbacks and difficulties of closeness centrality must be considered.

1.Disjoint multipath closeness centrality: In contrast to alternative paths, traditional closeness centrality simply considers the shortest paths. discontinuous multipath closeness centrality is a new metric that takes into account multiple shortest and discontinuous quasi-shortest paths to discover nodes that are multiply connected and close to all other nodes. Better-connected nodes are identified by this metric, which also retains strong agreement with other closeness measures. [39] Closeness of disjoint multipath A new measure called centrality was created to evaluate a node's centrality in a network by accounting for numerous shortest paths as well as discontinuous quasi-shortest paths in addition to the shortest paths. This method yields a more complex measure of closeness by identifying nodes that are multiply connected in addition to being well-connected.

2. Distinctiveness centrality: This is a novel measure that gives nodes that maintain connections with the network's perimeter a better score. It finds social players with more distinctive network relationships and penalizes links to highly connected nodes. While they employ different strategies, closeness centrality and distinctiveness centrality both aim to identify significant nodes within a network. Distinctiveness centrality adds a dimension of uniqueness by prioritizing nodes that maintain distinctive connections, especially with the network's perimeter, while closeness centrality stresses efficiency and centrality in the network. The delicate balance between being important and unique determines how the two relate to one another.[36]

Degree centrality and closeness centrality have a nonlinear relationship in which the logarithm of degree determines the inverse of closeness.[35]

Considering nodes as processes, input, and output betweenness centralities are suggested as measures to identify significant nodes in directed networks.[40]

Effective calculation: Developing methods that split large networks into biconnected components and employ incremental update approaches to maintain computation results when graphs change is one way to solve the time-consuming nature of accurate closeness centrality calculations.[38] Subgraphs that are maximally biconnected within a graph are called biconnected components. Stated differently, it is a connected subgraph that maintains its connectivity even if any vertex (node) or edge is eliminated. In graph theory, biconnected components are essential, especially when it comes to recognizing strong network architectures. [41]

Dynamically developing directed and weighted networks are those that constantly add new nodes and edges as they develop. The connections between nodes in this class of networks are given a weight and a direction. According to the theory of dynamic growth, the network grows when new nodes and edges are added, including modifications and additions to the system. A fast approach that adjusts to the dynamic structure of the network is used to compute closeness centrality values in dynamically developing directed and weighted networks. This method optimizes the closeness centrality computation without having to do a new calculation by accounting for dynamic changes like the insertion of additional nodes and edges.

1.2.4 Eigen Vector Centrality Measure

Eigenvector centrality is the term used to describe the primary eigenvector of a graph's adjacency matrix. [42] It is a metric for determining a node's relative relevance in a network based on the quantity of interactions it has as well as its structural placement. Eigenvector centrality has been extensively employed in numerous applications, including multiplex network characterization and node ranking in complex systems.[43] To overcome some of its shortcomings, like the localization of the major eigenvector and challenges with centrality weight assignment, it has also been expanded upon and altered. Overall, eigenvector centrality is a useful technique for determining the importance of nodes in complicated networks.[44]

It considers the quantity of interactions a node has in addition to its structural placement within the network.[42]

An effective metric for determining node relevance in intricate networks is eigenvector centrality. It has been expanded upon and used in a variety of contexts to provide insights on network dynamics and structures. It considers both the quantity of interactions and the structural locations of nodes.

The primary eigenvector of the graph's adjacency matrix serves as its foundation. The following are the main characteristics of the eigenvector centrality mathematical formulation:

The eigenvector centrality (EC) for a node i in a graph with adjacency matrix A

$EC(i) = \frac{1}{\lambda} \Sigma_j A_{ij} \cdot EC(j)$

where:

$EC(i)$ is the eigenvector centrality of node i

A_{ij} represents the element at the ith row and jth column of the adjacency matrix A

λ is the eigenvalue corresponding to the eigenvector centrality.

Weights can be added to the diagonal elements of strongly connected components in the adjacency matrix to use the principal eigenvectors of the adjacency matrix as centrality measures for digraphs.

For layer-coupled multiplex networks, where the layers represent various relationships and interactions between entities, matrix function-based centrality measures have been extended. By considering the graph's adjacency matrix and utilizing the principal eigenvectors of the matrix or modified matrices based on various graph components, eigenvector centrality may be computed.

1. Start with the adjacency matrix of the graph. The graph's nodes' connections are represented by this matrix.
2. The adjacency matrix is symmetric in the event that the graph is undirected. The adjacency matrix of a directed graph could be asymmetric.

An eigenvalue issue must be solved to determine the eigenvector centrality. The formula to be solved is $A_v = \lambda_v$ where,

A is the adjacency matrix;v is the eigenvector;λ is the corresponding eigenvalue.

The adjacency matrix's primary eigenvectors are calculated. The biggest eigenvalues are represented by these eigenvectors. Next, the acquired eigenvector is normalized to make sure that all its members have a magnitude of one or add up to one. It is important to do this normalizing step when comparing centrality ratings between various nodes. The graph's nodes' centrality scores are represented by the components of the normalized eigenvector. Nodes in the network are regarded as more central when they have greater centrality scores.

The following algorithms are used to compute eigenvector centrality:

It is a well-established and effective technique that is frequently employed in many different applications, including PageRank, spectral embedding, and principal component analysis. When the right parameters are selected, the power method can be further enhanced by utilizing variations such parameterized power methods, which can lead to more efficient iteration techniques. For the power iteration approach to be optimized, choosing the right parameters is essential. It entails thinking about starting points, shifts, normalization, parallelization, iteration limitations, convergence criteria, and restarting. When working with huge matrices or in situations where the dominating eigenvalue is surrounded by others, modifying these values can result in faster and more consistent convergence.

For a given graph G= (V, E) with $|v|$ vertices let A = $(a_{v,t})$ be the adjacency matrix, where $a_{v,t} = 1$ if the vertex v is linked to vertex t, and $a_{v,t} = 0$ otherwise. The relative centrality score x_v, of vertex v can be defined as:

$$x_v = \frac{1}{\lambda} \sum_{t \in M(v)} x_t = \frac{1}{\lambda} \sum_{t \in V} a_{v,t} x_t$$

Where $M(v)$ is the set of neighbours of v and λ is a constant. This may be represented in vector notation as the eigenvector equation with minor changes.

$$Ax = \lambda$$

Through the construction of a three-order tensor known as the two-step's tensor, eigenvector centrality can be expanded to take into account a node's first and second neighbors.

Localization phenomena are displayed by eigenvector centrality in networks that are easily partitionable through the removal of a vertex cut set. Due to its sensitivity to the overall network structure and its iterative computation process, which reinforces centrality within well-connected communities, eigenvector centrality is a useful tool for identifying localization phenomena in networks, which are characterized by concentrated centrality in specific regions.

Different features of node importance in networks can be captured by formulating eigenvector centrality in different ways, such as taking neighboring nodes, directed networks, and multiplex networks into account.

It is possible to use eigenvector centrality to take into account a node's influence inside a particular subnetwork. Eigenvector centrality is a sophisticated and context-aware measure of centrality that goes beyond node degree, making it an excellent choice for capturing a node's influence in a subnetwork since it naturally takes into account both direct and indirect connections. This makes it possible to comprehend node relevance in various situations more deeply.[45]

A different variation is the one-step extrapolation method, which is especially useful for issues with tiny spectrum gaps as it speeds up convergence to the dominating eigenvalue/eigenvector pair. [46]Furthermore, a variation known as the Adaptive Power Method (APM) allows for reliable estimation of dominant eigenvectors even with partial observations. APM adaptively picks a subset of a matrix's entries to observe based on the current estimate of the top eigenvector.

Using the primary eigenvectors of modified matrices based on several strongly connected components in a digraph is another technique. By adding data from nodes that are two steps away, the two-steps eigenvector centrality can be seen of as an extension of eigenvector centrality. The network's unique properties and the node influencing factors that must be taken into account determine which of these indicators to use. Directed Graphs: The method is intended just for directed graphs, also known as digraphs, in which directionality is present in the edges. A unique way to analyze centrality within substructures of directed graphs is to use the major eigenvectors of modified matrices based on strongly linked components.

The interpretation of high and low eigenvector centrality scores might provide insight into the relevance of nodes in a network.

1. Nodes with high eigenvector centrality scores are those that are linked to other nodes that are likewise very central, in addition to being well-connected. These nodes are important for the network's information flow and are regarded as influential.[42]

2. Nodes with low eigenvector centrality scores are thought to be less prominent and have fewer connections throughout the network. These nodes might not be very important in the spread of information and might only have restricted access to it.[42]

3. Eigenvector centrality can be extended and adjusted to capture different characteristics of node importance in other network structures, such as directed networks and urban networks with specialized data.[47][48]Beyond the conventional eigenvector centrality measurements, these extensions offer further insights into node importance.

Eigenvector centrality offers a node relevance measure that considers the connections made by the node as well as the significance of those connections. In order to comprehend the importance and influence of nodes inside a network, this statistic.

Understanding eigenvector centrality's benefits, drawbacks, and complementarity requires the following insights:

Similar to how β-centrality generalizes eigenvector centrality, Bonacich's β-centrality and related measures have been extended for directed networks. Based on the idea of eigenvector centrality, a centrality measure for multiplex networks has been developed, accounting for the various connections amongst nodes in each layer. Although it is difficult to estimate a network's eigenvector centrality only from nodal data, it is possible to do so by taking advantage of the spectral characteristics of graph filters.[43].

Bonacich's β-centrality is the new multiplex network index that is stated. Yes, it is an adjustment to eigenvector centrality. Based on the idea that links to nodes with high scores add more to a node's score, eigenvector centrality calculates a node's relative importance in a network.

In a network, the eigenvector centrality x_i of a node i is determined using the following formula:

$Ax = \lambda x$

where: A is the adjacency matrix of the network.

x is the eigenvector corresponding to the largest eigenvalue λ

Bonacich's β-centrality introduces a layer of complexity to the centrality measure by extending this concept to include the impact of a node's neighbors and their connections. It has been applied to directed networks, and associated metrics involve several adjustments to the original formula to take directed edge behavior into consideration. In directed networks, for instance, a node's influence is determined by both nodes pointing to it and by connections to other nodes that score well. To capture the influence flow in both entering and outgoing directions, the formulation must be modified as part of the extension. Iterative computation is required to calculate Bonacich's β-centrality.

$$x_i = \frac{1}{\lambda-\beta}\sum_{j=1}^{n} a_{ij}x_j$$

Where: x_i is the β -centrality score for node i;

λ is the largest eigenvalue of A.

The relationship between a node's centrality and that of its neighbors is iteratively updated until convergence. Similar to the power iteration method utilized for eigenvector centrality, the relationship with eigenvector centrality resides in the iterative process. On the other hand, Bonacich's β-centrality introduces the parameter β, which permits a more sophisticated regulation of the impact of neighbors.

A graph filter is a mathematical tool that allows you to manipulate signals on a graph. It is difficult to estimate eigenvector centrality in the context of networks since it depends on the overall structure of the network and is difficult to estimate from nodal data alone. Graph filters make use of the spectral properties of the graph by examining the adjacency matrix of the graph's eigenvalues and eigenvectors. This method's benefit is that it can estimate centrality even in situations where the complete network structure is unavailable, which increases its adaptability to situations with small data.

The new degree-like centrality method known as Weighted Symmetric Nonnegative Matrix Factorization (WSNMF) permits the exclusion of specific dyads from centrality calculations. It can be viewed as a generalized version of the current eigenvector-like centrality and employs the WSNMF approach. It has demonstrated effectiveness across a number of datasets. A pair of nodes (or vertices) and the link (or edge) connecting them are referred to as a dyad in network theory. It stands for the most fundamental component of a network's connectivity. Not Included in Centrality

Estimates: Because WSNMF allows for the exclusion, some node pairs and the linkages that link them might not be included in the centrality scores that are computed. This adaptability gives the centrality metric an extra degree of personalization. [49]

Extension of eigenvector centrality: By taking into account a node's first and second neighbors, the two-steps eigenvector centrality offers a deeper understanding of node importance than eigenvector centrality alone.[42]

Extension to directed networks: By taking into account the in- and out-centrality of nodes and their neighbors, Bonacich's β-centrality and related measures extend eigenvector centrality to directed networks.[47]

Localization phenomena: Eigenvector centrality exhibits localization phenomena in particular network configurations, such as hub node localization and other sorts of localization.[50]

The major eigenvector's localization and delocalization may make it challenging to allocate nodes centrality weights based on eigenvector centrality.[51]

Complementarity spectra: These can be obtained by understanding complementarity eigenvalues and reveal information about the structure of a graph.[52]

Better centrality metrics: To solve issues with directed and weighted networks, new centrality measures, such bow-tie centrality, have been proposed as extensions of eigenvector centrality. An modification of eigenvector centrality designed specifically for directed and weighted networks is called bow-tie centrality. In order to account for the complexity brought forth by directed edges and weighted connections, it computes centrality scores by utilizing the dominant eigenvector of an applicable matrix. [53]

While eigenvector centrality can be advantageous in terms of adding more information and generalizing to directed networks, there are drawbacks associated with localization phenomena and challenges in determining centrality weights. Additional options to investigate node relevance in complex networks are provided by complementarity spectra and enhanced centrality measurements.

An effective metric for determining node relevance in intricate networks is eigenvector centrality. It has been expanded upon and used in a variety of contexts to provide insights on network dynamics and structures. It considers both the quantity of interactions and the structural locations of nodes.

2. Centrality Indices

2.1 Introduction

Mobility services in large cities depend on underground transportation networks, thus it's critical to identify the key players and comprehend what happens to them when they malfunction. [54][55]

Based on the amount of information and study objectives, the algorithm analyses several approaches to correlate a graph with an underground network.

The Path Rank algorithm provides a scientific technique for determining the relevance of nodes in subterranean transportation networks, assisting in the development of focused management measures and the stable operation of urban rail transit systems.[54].

The Path Rank method, by ranking nodes based on their contribution to network pathways and their associated weights, is particularly effective in the fields of information diffusion and road data generalization. Information diffusion refers to the spread of information, ideas, or trends through the network, where nodes with higher Path Rank scores are typically more influential in propagating this information. Road data generalization, on the other hand, involves simplifying complex road network data into more manageable and understandable forms, where the Path Rank method helps identify key routes and junctions that should be prioritized in the simplified model [54]

2.2 Path Rank

Path Rank is a centrality Measure that ranks nodes in undirected networks according to the number and weight of pathways in the graph. It is the PageRank algorithm expanded upon. Path Rank is useful in the context of underground transportation networks because it may be used to mimic the responses of these networks' most crucial components in the event that they malfunction. To keep the mobility service at a suitable level, this is essential. A reference dataset was created by mapping 34 underground networks from cities around the world into graph representations. Path Rank was then applied to this dataset, offering a detailed analysis of node centrality within these networks. As highlighted in [74], the use of Path Rank not only facilitated the ranking of nodes based on their centrality but also proved essential in identifying key nodes that play a crucial role in the network's overall structure and efficiency.

Path Rank is built on mathematical concepts that give probabilities to nodes according to how far apart they are from one another.

Nodes in undirected graphs are ranked using the Path Rank method, a novel centrality measure, according to the number and weight of their paths. It assesses a node's relevance in complicated networks and is an Inspiration of the PageRank method.

Different methods for connecting a graph to an underground network: Graphs can illustrate underground networks in a variety of ways, depending on the amount of detail and study objectives. This variation in graph representation can have an impact on the use of Whatever Centrality Measure in investigating underground networks. The number and weight of pathways, node and edge weights, and distance scaling must all be considered while analyzing these networks. [54]

The Path Rank algorithm's technical details are as follows:

Now, we explain how PathRank is computed at each node, The number and weight of paths in the graph are used to determine a node's score in Path Rank.[56]

Let G= (V, E) given.

Assuming G has n nodes, designated as 1, 2,, n, we set x_i to be the score of node i in order to fix ideas. In PageRank, the neighbor nodes' scores determine the score in x_i the following way:

$Path\,Rank\;(j) = \Sigma_{i=1}^{n} b'_{ij}$ Where, b'_{ij} to be the sum of each path's contribution over all pathways from i to j, n is the number of nodes in the network.

2. Undirected Graph Adaptation: When evaluating centrality in undirected networks, the PageRank method may be changed to account for node distance. This modification takes into account translation possibilities dependent on the distance between nodes, allowing for a more personalized measure of centrality. There are also unique techniques, such as Set Push, that are especially developed for calculating single-node PageRank on undirected networks with reduced temporal complexity.[57][58]. In short, the fundamental distinctions between the PageRank method and its modification for calculating centrality in undirected networks are distance consideration and the creation of specialized algorithms for more efficient calculation.

Path Rank has the following advantages and strengths as a centrality measure for underground networks:

1. Considering other pathways: Path Rank considers all paths in the network in addition to the shortest ones, which is crucial for preserving network connection.[59]

2. The ability to rank nodes according to path characteristics: Path Rank enables a more thorough evaluation of node relevance in Underground networks by ranking nodes according to the quantity and weight of pathways.[54]

3. Flexibility and efficiency in diverse types of networks: Path Rank has been used to a variety of real-world networks, including transportation networks and urban street networks. [57] Overall, Path Rank provides a helpful technique to evaluate centrality in underground networks by taking into consideration both the quantity and weight of paths, as well as alternate paths for sustaining network connection.[54][59]

2.3 Icentr

The Icentr Centrality Index is a new centrality index that ranks nodes in a network using a mix of node and edge weights that are scaled depending on their distance from each other. It is especially relevant in the context of underground transit networks, which are critical for maintaining mobility services in major cities. The Icentr Index has been applied to 34 underground networks in cities throughout the world, creating a reference dataset for researching the most critical components of these networks and modeling their responses when they fail. Researchers may use the Icentr Index to study the centrality of nodes in underground transportation networks and get insight into their relevance and functionality.[54]

To identify prominent nodes in networks, several centrality metrics such as Betweenness Centrality, Closeness Centrality, and Degree Centrality have been utilized. [18]

The Icentr Centrality Index incorporates nodal data, takes into account node and edge weights, and may be used to rank nodes in weighted networks. It gives a complete measure of network node centrality.[4][18][60]

It belongs to the class of path-based centrality measures, which make use of information about the pathways from a given node to other nodes and geodesics between other nodes that contain a particular node. [4] The Icentr centrality index was mathematically studied.[61] .

Overall, the Icentr centrality index provides a theoretical framework for determining the relevance of network nodes based on their position and connectedness.[47][61][4]

The relevance of determining node importance in these networks rests in maintaining a desirable degree of mobility service and understanding the network's behaviors when components fail. Engineers may use the Icentr Centrality Index to identify the most significant components of subterranean transportation networks and model their reactions under various situations.

The procedure for calculating the Icentr centrality index for a given network is as follows:

As previously, we use an undirected simple graph G = (V, E) that represents the network under investigation, with nodes and edges equally weighted. To use fixed notation, the nodes are 1, 2,..., n, and the edges are e_1, e_2,...,e_r .We recall that an edge is a set with two unique nodes, and two distinct edges have only one common at most. Each node on an edge is a neighbor of the other node. The concept of neighbor is central to the definition of Icentr. The weights of the nodes are x_1, x_2,..., x_n, whereas the weights of the edges are w_1, w_2,..., w_r.

We note that if the graph is unweighted, all node and edge weights are equal to 1, or there may exist x, w such that x_i = x, w_j = w for every i = 1,..., n, and every j = 1,..., r. Nodes and edges are evenly weighted in this situation. It is possible that the nodes are uniformly weighted but the edges are not, or that the edges are uniformly weighted but the nodes are not. In general, neither nodes nor edges are weighted evenly. Finally, we note that the values assigned to each node by Another, not the Centrality index can be used as weights for nodes.[54]

Methodology: We Assume G is Connected, since the Icentr value depends only on the connected component containing the node. Let i_0 represent the node of interest. We partition nodes and edges into levels using a modified Breadth-First Search technique, as shown below. The single level 0 node is i_0. The neighbors of i_0 are the level 1 nodes, which are i11, i_{12},..., i_{1n1}. The level 2 nodes are i_{21}, i_{22},..., i_{2n2} neighbors of the level 1 nodes that are not in a previous level. In general, a node is in level h if it is a neighbour of a node in level h-1 but not of a node in level k for some k < h-1. The level h nodes are i_{h1}, i_{h2},..., i_{hnh}. No node belongs to more than one level. Furthermore, because we are assuming that the graph G is connected each node belongs to a level. To split edges into levels, consider that an edge e ={ i, j }joins two nodes in either distinct or the same level. In the first scenario, the nodes are from successive levels. Moreover,

we set $lev(e) = \max\ (lev(i), lev(j))$

In the second scenario, we set.

$lev(e) = 1 + lev(i) = 1 + lev(j)$

This selection of edge levels corresponds to the sequence in which they can appear in a path beginning with i_0. In reality, if an edge links two nodes at different levels and is on a path from i_0 , it is at the location corresponding to the edge's maximum level. If, on the other hand, an edge links two nodes in the same level and is on a route from i_0, it is at the location corresponding to the next level with respect to the nodes in the edge.

We recall that w(e) is the edge weight e. Let x(e) be the weight of the node at the highest level in e if e links two nodes at different levels. The contribution of e to the value Icentr takes at i_0 is then calculated.[54]

$$ic\ (e) = \frac{x(e)}{2^{lev(e)-1}}\omega(e)$$

If e= { i, j} links two nodes at the same level with weights x_i, x_j, the contribution of e to the value Icentr takes at i_0 is then $iC(e) = \frac{x_i+x_j}{2^{lev(e)}}\omega(e)$

Because the total value is simply the sum of the separate contributions, we have.

$$\text{Icentr}\ (i_0) = \sum_{j=1}^{r} ic(e_j).$$

Here are the benefits and strengths of the Icentr as a centrality metric for underground networks.

1. Appropriateness for rating nodes in underground transportation networks: Underground networks are critical for keeping cities moving, and the Icentr is particularly built for rating nodes in these networks.

2. Consideration of several methods for associating a graph with an underground network: The approach used to associate a graph with an underground network takes the degree of detail and the goals of the study into consideration, allowing flexibility in evaluating different parts of the network. The Icentr provides a personalized way to ranking nodes in underground networks that takes into account the unique characteristics and relevance of these networks.[54]

The Icentr centrality index has various limits and obstacles when used in the investigation of Underground networks. Here are some crucial points to think about: The association of a graph to a transportation network is not unique, and depends on the degree of details and study objectives. This variation in network representation might be problematic when using centrality indices.[54]

Overall, the Icentr centrality index is a useful tool for analyzing network structures and identifying essential nodes across various fields.

2.4 Ishortest

Depending on the index taken into consideration, the centrality metrics seek to identify the nodes in a graph that are the most significant. Such nodes are those whose cancelation is likely to significantly alter the graph's performance, yet no one can ensure it. There is no straightforward method to compare the values of a centrality measure on a network before and after deleting a node and any edges that contain it.[62]

Its values are determined by a graph as well as all graphs created by cancelling one node at a time. To calculate Ishortest, we compare the lengths of the shortest pathways before and after deleting a node. Because the length of a path in graphs is clearly defined, whether weighted or unweighted, we can compute Ishortest for nodes in any graph.

Let G= (V, E) be a graph, with V representing the set of nodes and E representing the set of edges.

We Assume that G has at least three nodes.[62]

where we assume that the nodes are ordered so that $w(x, y)$ contributes to the total above just once. Now, we choose a node v and compute the total length of the shortest routes from V, that is, $w(v, G) = \Sigma_x W(x, v)$

Furthermore, we create the new graph G_V = (V′, E'), where V' = V-{v} and E' is the collection of edges in which v does not appear. The subgraph G_V, according to G and v, may stay connected or not they are related. The value of Ishortest at v is computed in different ways, in accordance with G_V's connectivity characteristics.

If we assume that G_V is connected, we compute $w(G_v)$, which is the total length of shortest routes in G_V , We set

$$Ishortest(v) = \frac{w(G_v)}{w(G) - W(v, G)}$$

We call it IS1 for convenience.

Of course, because V has at least three nodes, the denominator is strictly positive. Furthermore, if we consider two nodes in V', then $w(x, y) \leq w_v(x, y)$, where $w_v(x, y)$is the length of the shortest path in G_V from x to y, since certain edges in G are no longer edges in G_V.

As a result,$w_v(G_v) \geq w(G) - w(v, G)$ [62]

In other words, Ishortest(v) ≥1

However, we now suppose that Gv contains two or more connected components, and hence v is a cut-vertex, as these nodes are named Of course, there is no path from x to y when the two nodes belong to distinct connected components, therefore when the cut-vertex is removed, there is a loss of connections. Furthermore, when they are members of the same connected component, the length of the shortest path in Gv is greater than the length of the shortest path in G. Both the loss of connections and the increase in distance must be taken into account. As a result, assessed at a cut-vertex Ishortest is pair. We recall from the definition of betweenness that $\sigma(x,y)$ is the number of shortest routes from x to y in a graph. As a result, we define $\sigma_v(x,y)$ as the number of shortest routes between x and y in Gv:

If $\sigma_v(x,y) > 0$, the two nodes are in the same connected component; otherwise, $\sigma_v(x,y) = 0$. Otherwise, a good indicator of connection loss is

$$Ishortest_1(v) = \frac{\sum_{\sigma_v(x,y)=0} w(x,y)}{w(G) - w(v_1G)}$$

because the denominator is the entire length of routes that do not exist in Gv. IS21 is used briefly in the following sections.

Instead of evaluating the increase in overall length, we consider the sizes of the connected components that appear when v is eliminated. So we suppose Gv is the disjoint union of $G_1, \ldots., G_t$ and G_i contains n_i nodes. Of course, the number of nodes in is $n = n_1 + \cdots + n_t + 1$

Let

$$W_v(G_i) = \sum_{\substack{x < y \\ x,y \in G_i}} W_v(x,y)$$

$$W(G_i) = \sum_{\substack{x < y \\ x,y \in G_i}} w(x,y)$$

be the total length of the shortest pathways between nodes in G_v and G .

$$Ishortest_2(v) = \sum_{\substack{i=1 \\ n_i \geq 2}}^{t} \frac{n_i w_v(G_i)}{n w(G_i)}$$

The equation above represents the average increase in the lengths of the shortest pathways, weighted by the size of each connected component with at least two nodes. Finally, we established.

$$Ishortest(v) = (Ishortest_1(v), Ishortest_2(v))$$

Because the numerator is a addendum of the denominator, the first component is always less than 1. Of course, the larger the first component, the greater the loss of connections due to the node's cancelation. When there is one extremely big, connected component and a few little ones, we expect the second component to be near to 1. On the contrary, we anticipate that component to diminish when there are at least two connected components of equivalent size. In the case of weighted graphs, its value might be considerably greater than one, depending on the weights of the edges on the new shortest routes. It is worth noting that if (the cut-vertex) has degree two, this component is always equal to 1.[62]

3. Types of Risks Involved in Transportation Modes

Transportation modalities provide a variety of hazards and issues that must be handled in order to ensure efficient and safe operations. One critical aspect of evaluating the risks associated with transportation networks is the use of centrality indices, such as the Network Robustness Index (NRI), Path Rank, Icentr and Ishortest. Let's delve into a more detailed explanation of how these indices are calculated and their application in assessing transportation network resilience.

Transportation network interruptions or breakdowns are assessed using centrality indexes.

NRI (Network Robustness Index): The NRI is computed by evaluating the impact of individual network link closures on a road transportation network. This index uses weighted degree centrality to assess network resilience, with a focus on heavily travelled routes and connections to northern areas. The NRI is an important tool for calculating the impact of disruptions on network operations.[63]

Steps in Calculation:

1. Determine which road transportation network is being considered.
2. To evaluate the impact, select a network link.
3. Close the chosen link momentarily by simulating closure.
4. Analyze the impact of any changes to the network's performance, such as longer travel times or interrupted routes.
5. Repeat: To evaluate the overall impact, repeat the procedure for several links.

Compute NRI: The Network Robustness Index is determined by adding together each of the individual impacts.

The formula for NRI is as follows:

$$NRI = \sum_{i=1}^{n} \frac{wi}{W} \cdot \frac{di}{D}$$

w_i is the weight of the link i ; W is the total weight of all links: di is the degree of node i ; D is the total degree of all nodes.

Path Rank: Path Rank is a centrality measure in network analysis that is used to assess the relevance or influence of nodes in a network. Unlike other centrality measures that emphasize a node's immediate connections, Path Rank emphasizes a node's importance in facilitating various paths throughout the network. This metric is especially relevant in networks such as transportation or communication networks, where the importance of a node goes beyond its direct connections. Path Rank can provide insights into the network's robustness, efficiency, and vulnerabilities by focusing on paths.

Steps in Calculation:

1. The first step involves mapping out all possible paths within the network. This includes identifying all nodes and the connections (paths) between them.
2. In the case of transportation networks, each path is allocated a weight based on relevant parameters such as distance, capacity, or even traffic flow.
3. Calculate how each node contributes to the network's paths. This entails taking into account the quality and utility of the pathways that travel through or exit from the node.
4. Each node's Path Rank score is calculated based on its contribution to the overall path structure. The computation's details can change depending on the network's features and the weighting criteria employed.
5. Finally, nodes are ordered by their Path Rank scores. Higher scores indicate higher network importance or influence.

$Path\,Rank\,(j) = \Sigma_{i=1}^{n} b'_{ij}$ Where, b'_{ij} to be the sum of each path's contribution over all pathways from i to j, n is the number of nodes in the network.

Icentr: It is another centrality index that is weighted by the distance between nodes. It ranks nodes based on a combination of edge and node weights, providing information about node relevance in transportation networks. This measure is useful for assessing the impact of individual nodes on overall network resilience.

The following algorithm is used in the computation:

Steps in Calculation:

1. Determine the nodes, edges, node weights, and edge weights of the transportation network.
2. To determine a node's centrality, select it.
3. To find the Icentr value for the chosen node, use the computed sum.

   $ic\,(e) = \frac{x(e)}{2^{lev(e)-1}}\omega(e)$ where, $w(e)$ weight of the edge.

4. Go through the procedure again for every network node.

Icentr $(i_0) = \sum_{j=1}^{r} ic(e_j)$. Where, i_0 is the sum of the contributions from all edges connected to it, r is the number of edges connected to i_0

Ishortest:

Ishortest focuses on identifying nodes whose failure would have a significant impact on the overall performance of the network. It accomplishes this by assessing the nodes that, if disrupted, would cause significant changes in the network's functionality.

Steps in Calculation:

1. Draw a network of transportation nodes and edges.
2. Select a node to evaluate its significance.
3. Find the shortest routes between the chosen node and every other node in the network.
4. Apply a formula to determine the selected node's Ishortest value.
5. Go through the procedure again for every network node.

$$Ishortest(v) = \frac{w(G_v)}{w(G) - W(v, G)}$$

Natural catastrophes, infrastructure failures, accidents, and operational interruptions are all examples of transportation-related hazards.

Natural disasters: Transportation infrastructure is vulnerable to both natural and man-made severe events, which can cause malfunctions and interruptions. These incidents endanger the operation of transportation networks and can result in infrastructure damage and service interruptions.[64]

Infrastructure breakdowns: Infrastructure failures can provide risks such as a lack of punctuality, dangers to people, equipment, and cargo, and a reduction in the quality of logistical services. These failures have the potential to reduce the efficiency and safety of transportation networks.[65]

Accidents and operational disruptions: Transportation-related accidents, particularly those involving hazardous goods, can have serious effects for individuals, the environment, and property. events or events can also have an influence on the regular operation of transportation networks, causing interruptions in routines and work accessibility. [66]Understanding and controlling these risks is critical for guaranteeing transportation systems' resilience and safety. Risk assessment and management frameworks can aid in the identification and mitigation of these threats.[64]

The Performance of transportation networks and their ability to withstand interruptions have been the subject of much research in recent years. Transportation networks that are resilient are essential for enduring and regaining operations after disruptive incidents.[67] The resilience of transportation networks has been measured using a variety of definitions, measures, and techniques. [68] Resilience has been measured and improved via the development of measures, mathematical models, and techniques.[67] Some of the factors influencing the resilience of transportation networks include the features of the road networks as well as the management and organizational aspects.[69] Research has also been done on the resilience of public transportation networks, concentrating on the effects of disruptions and ways to mitigate them.[70]A composite resilience score is a useful tool for quantifying the effects of interruptions on the resilience of fast transit. Resilience evaluations have difficulties due to uncertainty and a lack of validated evidence. All things considered, this field's study offers insightful information for managing, planning, and designing transportation infrastructure to increase resilience.

It is crucial to evaluate the network's resilience to unfavorable occurrences for several reasons. Maintenance and dependability: Network service providers strive to maintain acceptable network performance in order to lower subscriber attrition. Resolving anomalies and keeping an eye on user experience on the network may help prevent poor network service levels and save operating expenses. The pace at which remedial action is implemented in response to network disruptions or events that affect the network service level is referred to as network resilience. Network resilience assessments assist operators in mitigating poor network performance and restoring acceptable levels. Evaluating the network's resilience to unfavorable occurrences is essential to guaranteeing dependable network performance, reducing downtime, and effectively resolving network issues.

Using fictitious scenarios, let's examine instances of how centrality indices are used to evaluate the resilience of transportation networks:

Network Robustness Index (NRI):

Situation: Network Robustness Index (NRI) Imagine a metropolis with an intricate system of road transportation. The network's ability to withstand possible disturbances is evaluated using the Network Robustness Index.

Utilization:

1. Finding the Important Links

2. Determine the importance and volume of traffic while choosing important road linkages.
3. Play out the closing of every single link separately.
4. Analyze the effects on travel time, traffic, and the accessibility of alternate routes.

NRI calculation:

1. Combine the various effects of connection closures.
2. To measure the overall robustness of the transportation network, compute the Network Robustness Index.

Planning Strategically:

1. Make infrastructure upgrades a priority by using the NRI results.
2. Determine which links would most significantly affect the operation of the network if they were disrupted.

Importance of Results:

1. The transportation network's total resilience is measured by the NRI.
2. A network with a high NRI is robust and has connectivity that would only cause minor disruptions.
3. Improving vital linkages first will increase the overall resilience of the network, which is significant.

Path Rank:

Scenario: Let us contemplate a city with an intricate public transit network. In order to improve network performance, Path Rank is used to determine which transit nodes are more important.

Application:

Analysis of Weighted Paths:

1. Analyze the number and caliber of routes connecting the various transit nodes.
2. Think about things like transfer efficiency, passenger volume, and connectivity.

Node Position:

1. Sort transit nodes in the network according to the weighted pathways they contribute.

Path Rank computation:

1. To give each transit node a score, apply the Path Rank algorithm.

Optimal Performance:

1. To determine the important nodes that, when optimized, would improve the overall performance of the transportation system, use the Path Rank results.
2. Invest in high-ranking nodes' infrastructure upgrades, such as more platforms or better transfer facilities.

Importance of Results:

1. Key transit nodes essential to the overall operation of the system are identified by Path Rank.
2. High-ranking nodes could be indicative of busy transfer hubs or densely populated passenger locations.
3. Infrastructure optimization at these nodes is important since it raises the overall transit system's efficiency.

Icentr:

Scenario: Let's say there is a region with a varied transportation system that includes both roads and trains. Icentr is used to evaluate a node's importance in sustaining effective connectivity.

Utilization:

Compute Node Importance:

1. Choose a transportation hub, such as a train station or junction.
2. To find the weighted sum for the node, take into account the weights of the edges and nodes, adjusted for distance.

How to calculate Icentr

1. To calculate the chosen node's centrality, use the Icentr formula.

Functional Importance:

1. Higher Icentr values for nodes signify increased relevance in preserving network efficiency.
2. For targeted enhancements like improved rail connectivity or optimized traffic flow, concentrate on nodes with high Icentr.

Importance of Results:

1. Operationally, nodes with higher Icentr values are more important for preserving network efficiency.
2. For targeted enhancements like improved rail connectivity or optimized traffic flow, concentrate on nodes with high Icentr.

IShortest:

Imaginary Situation: Let's say there is a city with an extensive urban road system. The Ishortest path algorithm is used to determine which nodes are crucial and whose failure would significantly affect network performance.

Utilization:

Quickest Route Evaluation:

1. Determine the shortest routes between each road intersection and the other connections in the network.

Computing the Ishortest:

1. To award scores to each node according to its significance in the network, use the Ishortest algorithm.

Risk Mitigation:

1. Determine which nodes are crucial and prone to disruptions by looking for those with high Ishortest values.
2. At these crucial nodes, implement focused risk mitigation techniques like better traffic control or upgraded infrastructure.

Importance of Results:

1. High Ishortest value nodes are crucial locations that are prone to failures.
2. At these crucial nodes, implement focused risk mitigation techniques like better traffic control or upgraded infrastructure.

The robustness and vulnerability of a network are crucial elements to take into account when assessing the resilience of intricately interconnected systems, such communication, and infrastructure.[71]

Numerous methods have been put forth to evaluate network robustness, such as gauging the resilience of resilient nodes to assaults and assessing how attacks influence the emergent connection of impacted nodes. [72] Moreover, a number of metrics have been created to measure network susceptibility, such as comparing the original network to the analysis of the network's decline in resilience. Additionally, spectral measures have been developed to evaluate network resilience, such as the second spectral moment. These measurements and methods offer insightful information about assessing the resilience and susceptibility of networks, which helps create efficient defense plans.[73]

The process of assessing different criteria and approaches is necessary to assess the risk of performance degradation in transportation networks. Standards for evaluating performance indicators for public transportation providers that include pertinence, efficiency in terms of both money and time, and quality of service.[74]

The performance assessment methodology emphasizes how crucial efficacy and efficiency are to public transportation systems. Effectiveness is the degree to which predetermined tasks are completed, whereas efficiency is the proportion of resources utilized to outcomes. There are distinct metrics for both commercial efficiency and production.[75]

An approach known as multi-dimensional evaluation is put out to evaluate the sustainable performance level of road freight transport in terms of environmental, and social aspects. It seeks to enhance supply chain and logistics management and offers a comprehensive performance evaluation.[76]. The process of evaluating performance takes into account a number of factors, including sustainability, efficacy, and efficiency. This method permits efficient management and enhancement of transportation networks as well as a thorough assessment of performance.

Centrality indices are a helpful instrument for calculating how much a disturbance affects the operation of the network. This is how they function:

Centrality measurements use a variety of criteria to assess a node's significance within a network. These metrics forecast the traits and significance of nodes inside the network.[1]Conventional methods of assessing centrality entail comparing the network's performance before and after a node fails. These methods, however, have drawbacks and might lead to irrational outcomes in some situations. A novel method has been put out to get over these restrictions, and it measures the network's residual performance following a node failure to determine the node's centrality. A more thorough knowledge of network interconnectivity is offered by this method. In conclusion, by analyzing node significance and tracking changes in network performance, centrality indices offer a quantitative method to evaluate the effects of disruptions on network operation.[77]

In transportation networks, centrality indices are useful instruments for evaluating the danger of performance degradation.

Metro disruptions: The association between passenger flows at metro stations and network centralities was investigated in a research on the Athens metro system using machine learning and linear regression models. The results can be utilized to support backup plans in case of disruptions and to calculate medium-term ridership estimates.[78]

Airport performance: An approach incorporating centrality measures such as degree centrality and betweenness centrality was used to develop a Global Airport Performance Score (GAPS) for ranking international airports. This methodology considers both traditional throughput criteria and air transport network topology to provide valuable insights for benchmarking airports.[79]

Distribution networks: Nodes with high degree centrality outperformed less central nodes, according to a research on the distribution network of a major German automaker. This demonstrates how network topology affects the performance of individual network nodes. [80] To sum up, centrality indices have been used to evaluate the risk of performance reduction in transportation networks, such as distribution networks, airport performance, and metro disruptions.

Various methodologies and frameworks may be used to address strategies and planning approaches for managing performance decrease risk in transportation networks.

The intensity and length of observed deviations from normal circumstances may be utilized to calculate operational interruptions in transportation networks using a quantitative multicriteria approach.[81]

PREP (Performance-based Resilience Evaluation Procedure): The PREP methodology generates equivalent resilience scores and may be used to any form of transportation infrastructure to aid in project prioritization, risk reduction, asset management, and infrastructure design for greater reliability.[82] Subsidy as a risk-mitigation strategy: Subsidies for railroad operators might encourage them to choose alternate routes away from high-risk areas of the network, minimizing the possibility of dangerous occurrences. [83] Transportation planners and operators may successfully manage performance decrease risks in transportation networks by implementing these techniques and approaches.

The importance of centrality indices in decision-making and risk management cannot be overstated.

Identifying prominent players: Centrality metrics aid in the monitoring of networked systems by identifying the most influential individuals. This is especially important in financial markets, where strong centrality measurements may be utilized to create generic indexes of financial institution centrality. Recognizing global risks: Centrality indices may be used to build statistical relationships among various risk categories, assisting governments and companies in understanding the importance of global hazards. These indices give useful insights into the links between economic, environmental, geopolitical, sociological, and technical concerns by quantifying the ambiguity around risk estimates.

The use of centrality indices to estimate the risk of performance decline may be integrated into urban planning and infrastructure management procedures in the following ways:

Identifying important stations and roads: The relevance of urban rail stations and road infrastructure may be assessed using centrality indices such as betweenness centrality and proximity centrality. These indices take into account aspects such as traffic characteristics and location advantage, assisting in the identification of critical stations and roads that have a substantial influence on the functioning of urban rail networks and emergency response routes.[84]

Evaluating the carrying capacity of functional urban infrastructures: Centrality indices may also be used to assess the carrying capacity of functional urban infrastructures. These indices can assist analyse the performance and use of urban infrastructures by taking into account elements such as load-carrier models and mean-variance analysis, allowing for optimal urban development and management.

Assessing urban performance and safety: Centrality indices can aid in the assessment of urban performance and safety. These indices may evaluate safety conditions in different cities and suggest opportunities for improvement in urban safety regulations by applying methodologies such as super-efficient data envelopment analysis (SE-DEA).[85] Decision-makers may get useful insights into the performance, significance, and safety of urban systems by incorporating centrality indices into urban planning and infrastructure management procedures, supporting informed decision-making and sustainable development.

Informed decision-making in transportation system resilience improves project prioritization, risk mitigation, asset management, and design while taking into account highway network features and user input for fair outcomes.[86]

The degree of risk in the transportation process may be estimated using real risk incidences along the route. This aids in comprehending any disturbances in the transportation process.[87] It is critical to assess these risks in order to provide effective risk management solutions. Models of risk analysis for risky commodities transportation are critical for decision-making and management.

4. Application of Centrality Indices in Risk Analysis Across Transport Modes

4.1 Introduction

When assessing risk in transportation networks, centrality indices are essential. The practical uses, advantages, and disadvantages of centrality indices are explored in this chapter, with an emphasis on how they might be used to evaluate risk, strengthen resilience, and improve the overall effectiveness of transportation networks.

4.2 Applications in Various modes of transportation

4.2.1 Road Networks

Identification of critical road segments vulnerable to disruptions:

Transportation planning and management must include the identification of key route portions that are susceptible to disruptions. Numerous techniques have been put out to evaluate the susceptibility of road networks. One method is evaluating each section in a network to determine how vulnerable a road segment is, then creating diversion routes to rejoin damaged segments.[88] A different approach identifies crucial road segments by combining various network studies, including betweenness centrality, road density, and road segment length.[89] A new geo computational technique that takes into account the effects of non-road landscape characteristics on traversability has been created to identify crucial road segments in a post-disaster setting.[90] These techniques offer useful data for traffic planning, emergency response, and strategic urban planning.

Case Studies and Examples:

In a real-world case study, centrality indexes have been used in risk assessments for road transportation.[91] The study aimed to determine the environmental susceptibility to hazardous material road spills and to propose prevention strategies and emergency notifications.[92] The method entailed combining vulnerability analysis with accident data from a specific highway segment.[93] According to the findings, several incidents happened in locations of high susceptibility, [94] A geographic information system was used in the study to create risk management maps, which are necessary for alert systems and prompt environmental protection. The case study demonstrates the actual application of centrality indices in analyzing and managing hazards connected with dangerous material transportation along roadways. It has been used successfully in transportation planning to identify crucial road portions that are sensitive to disruptions. Scardoni and Laudanna created the concepts of centrality interference and centrality robustness, which allowed them to forecast the impacts of local network changes on single nodes as well as global network functionality. [95]Oliveira, Portugal, and Juliao proposed a conceptual framework for ranking the value of road network linkages based on congestion and vulnerability. They discovered that depending exclusively on congestion signs can result in incorrect solutions.

Global Applications

Various areas and countries have used centrality analysis to improve transportation planning and management. Hellervik et al., for example, show how to employ preferable centrality in the southern half of Sweden to examine the geographical distribution of consequences from transportation infrastructure expenditures.[96] Cheng et al. provide centrality measurements for the Singapore subway network that include transit time delay and commuter flow volume, providing insights for network design and dealing with failures.[97] Tsiotas and Polyzos propose mobility centrality as a measure for analyzing interregional road networks in Greece, emphasizing its capacity to represent flow trends.[98] Rubulotta et al. examine the relevance of centrality in sustainable mobility planning and propose a new accessibility measure, emphasizing the potential correlation and efficacy of centrality and accessibility indexes[99]

Utilizing centrality analysis for disaster planning and preparation.

In disaster planning and preparation, centrality analysis is used. It assists network operators in incorporating catastrophe event forecasts into their network operation plans.[100] Furthermore, after a disaster, centrality analysis can be used to determine which communication networks are more centralized in certain locations, which can help with organizing evacuation drills and community development.[101] Moreover, centrality techniques like Betweenness, Degree, and Closeness can be used to evaluate conceptual resources for crisis management and disaster risk reduction, promoting clear and simpler communication amongst experts across different areas.[102]

Estimation of road network resilience through complex network analysis.

It is possible to estimate the resilience of the road network by using complex network analysis. This involves determining the impact that component failures have on the functionality and overall performance of the system. By representing the relationships between road network components and the effects of failures, complex network analysis makes it possible to estimate the resilience of the road network.[103] Road network resilience has been measured using a variety of measures and indices, including a resilience index that includes demand- and topological-related data.[104] Road network resilience can be increased by updating network components, designing with additional capacity and redundancy, and reacting nimbly to interruptions.[105]

4.2.2 Rail Transit System

Evaluation of the importance of urban rail terminals using complex network theory.

The relevance of urban rail terminals can be assessed using complex network theory. A number of techniques have been put out to evaluate the significance of stations in urban rail networks. One approach considers the traffic patterns and structural features of the lines where the stations are situated and is based on complex network theory.[84] Another approach entails examining the rail transit system's global characteristics and topological structure, as well as locating subgraphs and motifs inside the network.[106] Furthermore, overall network importance and transfer waiting numbers can be used to objectively assess the significance of transfer stations and lines.[107] There are several measurements that can be used to determine the relative importance of nodes in urban rail transport networks, including node property, degree centrality, closeness centrality, betweenness centrality, and passenger flow centrality.[108]

Analysis of rail transit operation risks through centrality indices.

Risks associated with operating rail transit are examined using centrality indices. The network centralities of urban railway stations are determined using a variety of centrality criteria, including betweenness, and closeness. This can help to clarify the number of passengers and average trip time at each station.[109] Fault tree analysis is used to identify and assess risk variables for urban rail transit (URT) accidents, and a risk evaluation index system is built. [110] The Degree of Nodal Connection (DNC) index is a complete set of measures that is suggested to evaluate vulnerability of large heavy rail networks. Transfer stations and links are considered by the DNC index, from which four indicators of network vulnerability develop.[111] Advanced measures like travel patterns and passenger flows are employed in conjunction with typological methodologies for vulnerability analysis of public transportation networks, especially urban metro systems. Based on link centrality, serviceability is a dynamic approach that considers a variety of variables and shifts in the welfare of passengers.[94] In order to determine the significance of stations and lines, the centrality features of an urban rail network—such as degree-based, betweenness-based, and closeness-based indices—are examined. It is discovered that the closeness centrality is most significant to the line's operational state.[112]

Risk variables are factors or variables that contribute to the occurrence or severity of accidents in the context of urban rail transport accidents. Human error, equipment malfunction, infrastructure challenges, and environmental circumstances are examples of variables. After identifying the risk variables, the following step is to evaluate their relevance and possible impact on the occurrence and severity of accidents. This evaluation entails determining the likelihood of each risk variable leading to an accident as well as the potential repercussions if one does occur. Accidents involving urban rail transit systems such as subways, light rail, or commuter trains are referred to as urban rail transit accidents. These accidents can vary from simple mishaps to large disasters, with serious consequences for public safety and transportation infrastructure. In the context of urban rail transport accidents, fault tree analysis is used to systematically investigate the causes and consequences of accidents in order to identify and assess the risk variables involved. Appropriate actions can be implemented to prevent or lessen accidents in urban rail transit systems by recognizing the underlying elements and their possible impact. [113]

Case Studies and Examples:

Regression models have been created to demonstrate the relationship between passenger flow distribution and centrality indicators, allowing for the approximate calculation of passenger flow in public transportation networks.[94] Overall, evaluating centrality indices in rail transportation systems is critical for comprehending network features, optimizing network design, and making educated planning decisions. The study focuses on the Shanghai urban rail network, which is a complex and critical part of the city's public transportation system degree-based, betweenness-based, and closeness-based indices as three measures of an urban rail network's centrality. It compares the results of the investigation to the operational conditions of the Shanghai urban rail network. Closeness centrality is the most important indicator of the line's operating condition. The approaches used are the Degree-based index, the Betweenness-based index, and the Closeness-based index. The analysis of centrality features is useful for urban transit management and operations, the results showed that Degree Centrality is utilized to assess the potential travel activities at a station. The operational degree centrality of a station better represents the activities available to tourists.[114]

Study of risk propagation dynamics in rail networks.

The dynamics of risk spread in rail networks have been thoroughly investigated. The analysis of risk production, propagation, and control in urban rail transit (URT) systems has been done using a variety of techniques. Complex network theories and accident causality theories have been used to analyse the propagation path and law of URT hazards, with the goal of preventing and controlling operational mishaps.[115] The resilience of rail networks has been analysed using measures such as train and passenger delay minutes, showing the network's sensitivity to failures and the need of backup restoration during peak travel hours.[116]

4.3 Comparative Analysis of Centrality Indices

Strengths and Limitations by Transportation Mode:

This section will look at how the strengths and limits of centrality indices differ across different types of transportation modes—road networks and rail transit systems.

Road Networks:

Strengths: Centrality indices, such as betweenness centrality, are used in network analysis to determine the importance or centrality of nodes (in this case, road segments and intersections) inside a network. The degree to which a node is located on the shortest paths connecting other nodes in the network is precisely measured by betweenness centrality. This indicates that in the context of road networks, road segments or intersections with high betweenness centrality are those that are located on many of the shortest pathways between other road segments or crossroads. The utility of these centrality indices is demonstrated in the context of traffic analysis and emergency response planning.

Limitations: Traffic conditions are not constant and can change quickly. Variations in traffic volume, speed, and congestion levels are examples of these shifts. The indices specified in the highlighted text may be incapable of capturing these dynamic changes in real time. The indices may have been produced using assumptions and conditions that do not apply to all traffic scenarios. They may, for example, presume a constant traffic flow or uniform vehicle distribution, which may not be the case in constantly changing traffic conditions. The indices may be incapable of accounting for the interactions and dependencies that exist between various traffic measures. The indices' oversimplification of these patterns may result in incorrect judgments of traffic conditions.

Rail Transit Systems:

Strengths: The application of centrality indices to identify crucial stations and lines that are critical to a network's efficiency and resiliency. The research can determine the most important stations and lines in a transportation network by applying these indices. The study also underlines the importance of centrality indices in understanding passenger flow dynamics. The movement of passengers within a transportation network, including parameters such as the quantity of passengers, their origin and destination, and the routes they follow, is referred to as passenger flow dynamics. The research can acquire insights into how passengers move within the network and how the main stations and lines indicated by these indices affect passenger flow by employing centrality indices. This data can subsequently be used to make informed network planning, optimization, and resource allocation decisions. The Rail Transit system focuses on network efficiency and resilience in order to understand how the identified important stations and lines contribute to the overall performance and robustness of the network. The efficiency of a network relates to how efficiently it performs in terms of minimizing transit time, congestion, and delays. The ability of a network to endure and recover from disturbances such as accidents, natural catastrophes, or system failures is referred to as network resilience. The research can provide insights into how to increase the network's efficiency and resilience by identifying crucial stations and lines.

The use of centrality indices to pinpoint important stations and lines, the focus on comprehending the dynamics of passenger movement, and the attention to network resilience and efficiency are, all together, the research's strongest points. These findings may have practical consequences for transportation planning and management, assisting in the optimization of network performance and the overall passenger experience.

Limitations:

Due to the static nature of some centrality measures, there may be potential mistakes. The most important or influential nodes in a network are identified using centrality measurements. These measurements, however, are frequently generated based on a snapshot of the network at a certain point in time. This means that they ignore any changes or dynamics that may occur in the network over time. As a result, the centrality measures may not fully reflect the current condition of the network or any recent changes or disturbances.

Real-time disruptions and service changes discusses the difficulties associated with integrating real-time disruptions and service modifications. It is critical to evaluate the impact of real-time events such as accidents, road closures, or changes in service schedules while conducting transportation or network studies. Incorporating these real-time disturbances and changes into the study, on the other hand, can be difficult. It necessitates access to up-to-date and accurate data, as well as advanced modeling approaches to account for the network's dynamic nature. Failure to take these real-time aspects into account may result in erroneous or incomplete findings.

In summary the study has two drawbacks. To begin with, the static nature of some centrality measurements may lead to mistakes because they do not account for changes or dynamics in the network over time. Second, incorporating real-time disruptions and service changes into the study is difficult but necessary to ensure accurate and thorough results.

2.Comparative Effectiveness

This section ought to analyse the efficacy of multiple centrality indices across various forms of transportation, taking into account how each index delivers unique insights and has distinct implications:

Comparison Across Indices:

Indexes are mathematical measurements that are used in network analysis to quantify numerous aspects of a network's activity and structure. In network analysis, three indices are typically used: betweenness, closeness, and degree centrality.

Betweenness is a metric that quantifies the extent to which a node in a network is located on the shortest paths between other nodes.

1. It detects nodes in the network that serve as significant mediators or bridges.
2. Nodes having a high betweenness are critical for sustaining connectivity and allowing communication across the network.
3. Betweenness can be beneficial in identifying essential transit routes that connect different places or serve as major hubs in the context of transit routes.

Closeness is a measure that estimates how quickly a node may reach all other nodes in a network.

1. It identifies key nodes in terms of accessibility and efficiency.
2. Nodes with a high closeness can reach other nodes in the network faster, making them vital for efficient communication and information transfer.
3. Closeness can be used to analyse how quickly information or resources can spread via a network while measuring efficiency.

Degree centrality is a measure that quantifies the number of network connections a node has.

1. It detects nodes with a high degree of connectivity and a large number of direct neighbors.
2. Nodes having a high degree of centrality are critical for sustaining overall connection and encouraging interactions across the network.
3. Degree centrality can be used to determine which nodes have the most direct influence or impact on the network in terms of node importance.

Icentr to assess relevance, characteristics of edge and node weights are combined and adjusted by distance from each node.

1. This measure is used in transportation for assessing the importance of nodes based not only on their connection but also on the quality and capacity of connected routes.

Ishortest focuses on finding critical nodes that, if interrupted, have a major impact on network performance.

1. It is useful in risk analysis and resilience planning for transportation networks since it identifies nodes whose failure would have the greatest impact on network functionality.

4.4 Integrating Centrality Indices with other Risk Assessment Techniques

Integrating centrality indices with other risk assessment methodologies, such as the Network Robustness Index (NRI) and the Topological Vulnerability Index (TVI), can result in a more complete knowledge of network hazards and vulnerabilities. Each of these methodologies provides distinct information.

1. Integration with the Network Robustness Index (NRI)

1. NRI assesses a network's total resilience to interruptions, measuring how effectively the network retains functionality in the face of failures or attacks.
2. It frequently entails assessing network performance under various simulated failure scenarios.

Complementing with Centrality Indices:

1. The use of centrality indices can help identify crucial nodes whose failure would have the greatest impact on the network. Nodes with high betweenness centrality, for example, may be critical for network connection.
2. NRI can then focus on simulating disruptions at these nodes to determine how their failure might affect the overall network resiliency by using centrality indices to pinpoint crucial nodes.
3. This integration contributes to strategic network robustness by identifying and reinforcing crucial nodes or rerouting connections to mitigate the impact of their possible failure.

2. Integration with the Topological Vulnerability Index (TVI)

1. TVI evaluates a network's vulnerability to disruptions based on its topological properties, detecting susceptible points in the network structure.
2. It's used to figure out how network layout affects susceptibility and how to design for redundancy and resilience.

Complementing with Centrality Indices:

1. Centrality indices can improve topological analysis by identifying nodes and links that are not only topologically significant but also critical based on traffic flow, utilization, or other functional measures.

2. TVI analysis can be extended beyond simply topological structure to include factors of network utilization and relevance, providing a more comprehensive perspective of susceptibility.
3. The combination of TVI and centrality measures can be used to design more effective resilience solutions that focus on both the structural and functional components of the network.

3. Overall Integration Benefits

The combination of centrality indices with NRI and TVI enables a more comprehensive risk assessment that combines structural, functional, and usage-based aspects.

Resources can be allocated more efficiently to regions where they will have the most impact on strengthening network resilience by identifying both physically susceptible and functionally vital portions of the network.

Case Studies and Practical Applications

1. ***Underground Transportation Networks***

Case Study Overview: The underground Transportation networks uses innovative centrality indices, PathRank and Icentr, to underground transit networks, which are critical for urban mobility.
The emphasis is on identifying the most critical stations and tracks and simulating their reactions to probable disruptions.
Methodology and Application: The process entailed graphing and analyzing the underground networks of 34 global cities using the new indices. The approach sought to identify stations whose loss would result in the greatest reduction in network performance.
Outcomes: The use of these indices showed several aspects of the underground networks as well as crucial nodes important for maintaining service levels.

2. ***Maritime Transportation Network***

Case Study Overview: A network analysis of marine transportation, with container ports acting as nodes and shipping services acting as edges and to better understand the dynamics and characteristics of the maritime

transportation network, including the flow of products and services between ports.
Methodology and Application: The network topology was analyzed using centrality methods, with the extra feature of weekly transit capacity as edge weight.
Outcomes: The study provided insights into the structural value of specific ports and shipping routes, which will help guide risk management in marine trade.

3. *Air Transport Network of China (ATNC)*

Case Study Overview: In this scenario, classical centrality indices were applied to China's air transport network. Centrality indices can assist in identifying the most important airports in terms of connectivity, influence, or control over the network in the context of the air transport network.
Methodology and Application: It Included all cities with operational airports and used Degree, Closeness, and Betweenness centrality indicators over a six-month period.
Outcomes: The identification of airports crucial for network performance and economic impact was made easier by the strong link between centrality ratings and socioeconomic variables. This information is critical for risk management and planning for future air travel disruptions.

4. *Commuter Flow and Time Delay Analysis*

Case Study Overview: Cheng et al.'s study offered new indices concentrating on commuter flow and transportation network delays. The goal of this research was to develop new indices that focus on two distinct features of transportation networks: commuter flow and delays.
The study's goal was to gain a better understanding of how passenger flow and delays affect transportation networks, as well as to establish new measurements or indices to quantify these elements.
Methodology and Application: By measuring the effects of node disruptions on commuter flow and trip time, the indexes offered a more thorough understanding of network performance.
Outcomes: These indices allowed for a more in-depth understanding of how interruptions affect passenger experiences and network efficiency, which is critical for mitigating the risks associated with service outages.

5. ***Vulnerability Analysis in Transportation Networks***

   Case Study Overview: The investigation concentrated on many factors of network vulnerability, such as connectivity, accessibility, and capacity.
   Methodology and Application: Topology-based indices were used to quantify changes in network performance and the shortest distances between nodes during disruptive circumstances.
   Outcomes: The study provides an integrative approach to analyzing network vulnerability, assisting in resilience planning against a variety of disruptive events.

Finally, these case studies demonstrate that centrality indices are excellent tools for transportation network analysis, providing detailed insights into critical components and their roles in network functionality. Transportation planners and authorities can better foresee, prepare for, and respond to disturbances by using these indices, assuring the resilience and reliability of transportation networks in the face of a variety of obstacles. This strategic application of centrality indices is critical to ensuring continuous and efficient transportation services, which are required for modern civilizations to function.

Conclusion

Centrality indices are mathematical measurements that are used to determine the importance of nodes in a network. Centrality indices can be used in transportation risk analysis to identify significant nodes or areas in a transportation network that are more exposed to hazards or disruptions. Researchers can get insights into the possible implications of risks on the whole network and prioritize mitigation solutions by assessing the centrality of nodes.

5. Discussion and Proposals

The key arguments and findings in the preceding chapters of the thesis were around the application and significance of centrality indices in assessing and managing performance reduce risks in transportation networks. Here is a summary of these main points:

5.1 Overview of Key Findings

Importance of Transportation Networks*:* The first section of the thesis emphasized the importance of transportation networks in enabling the international movement of people, goods, and services. It highlighted that these networks are the foundation of contemporary society, and their performance is critical to economic and social progress.

Risks to Transportation Networks: The thesis highlighted a variety of transportation network risks, such as traffic congestion, accidents, natural catastrophes, and other interruptions. These threats have been identified as having a substantial impact on the dependability and functionality of transportation systems.

Role of Centrality Indices in Network Analysis: Degree centrality, betweenness centrality, closeness centrality was all thoroughly examined. These measurements have been demonstrated to be significant in identifying critical nodes and pathways in transportation networks, aiding in the understanding of the network's structure and behavior.

Application of Centrality Indices in Risk Assessment: We demonstrated how to use centrality indices to identify essential components in transportation networks. It is feasible to foresee regions of vulnerability and future performance decline by identifying these critical nodes and linkages. Different measurements of centrality were found to provide varying insights into the network's structure. For example, betweenness centrality aided in understanding node responsibilities in information flow, whereas closeness centrality shed light on the effectiveness of information dissemination across the network.

Enhancing Network Resilience: According to the results, centrality indices may be useful in strengthening the resilience of transportation networks. Transportation planners and decision-makers can implement preventative measures and establish effective contingency plans by knowing crucial nodes and linkages.

The usage of these indicators has been found to aid in proactive management by reducing the risks of performance declines and providing more strong and reliable transportation networks.

5.2 Proposals for Practical Application

Optimizing Traffic Flow:

1. Use centrality measures to discover critical nodes and routes that have a substantial impact on traffic flow.
2. To reduce congestion, propose targeted infrastructure improvements or traffic control measures at these important areas.
3. Incorporate real-time data analytics to dynamically change traffic signals and control flow based on current network circumstances.

Increasing Traffic Flow:

1. Use centrality measures to identify crucial nodes and routes that have a significant impact on traffic flow.
2. Propose targeted infrastructure improvements or traffic control measures in these critical regions to decrease congestion.
3. Incorporate real-time data analytics to alter traffic signals and govern flow dynamically based on current network conditions.

Improving Emergency Response Strategies:

1. Map ideal routes for emergency services using centrality indices to ensure timely access to crucial places.
2. Create emergency response plans that take advantage of the network's most connected and efficient channels.
3. Coordinate drills and training with local authorities and emergency services, leveraging network analysis to improve preparation.

5.3 Limitations and Future Research Directions

1. Dynamic Nature of Transportation Networks:

Limitation: centrality measurements sometimes presume a static network topology, which may not effectively reflect the dynamic nature of transportation systems.

Future Research: Create dynamic centrality measures that can adjust in real-time to changing traffic patterns, road conditions, and network configurations.

2. Computation and Complexity:

Limitation: The calculation of centrality measures for large and complex networks can be computational and time-consuming.

Future Research: Look into more efficient algorithms and computational tools, such as cloud computing or parallel processing, to better handle large-scale network analysis.

3. Sustainability and Resilience:

Limitation: There has been little research into how centrality indicators can improve the resilience and sustainability of transportation networks.

Future research: should look into the role of centrality in supporting resilient and sustainable transportation, particularly in light of climate change and urbanization challenges.

5.4 Case Studies and Real-World Applications

Case Study: Urban Traffic Management in Singapore

1. Background:

Singapore is a city-state in Southeast Asia. Singapore is a good example because of its sophisticated urban planning and traffic control systems. The city-state is known for its creative use of technology in traffic management and public transit.

2. Implementation of Centrality-Based Strategies:

Traffic Signal Optimization: Singapore identified crucial intersections and frequently used routes using centrality criteria for traffic signal optimization. The city then developed an adaptive traffic signal system, which dynamically adjusts light timing based on real-time traffic data.

Public Transportation Routing: Centrality analysis was also used to optimize bus routes. The public transportation system was reorganized to improve efficiency and reduce transit times by identifying critical transit nodes and high-usage corridors.

3. Outcomes and Improvements:

Traffic Flow: The adaptive traffic control technology improved traffic flow significantly, cutting travel times during peak hours.

Reduced Congestion: Route optimization based on centrality and traffic signal tweaks helped to reduce congestion, particularly in downtown areas.

Public Transportation Efficiency: With shorter wait times and more direct routes to high-demand locations, public transportation became more reliable and efficient.

Environmental Impact: Improved traffic flow and public transportation efficiency also helped to reduce automobile emissions, which aided Singapore's environmental aims.

This case study can provide vital insights into the actual application of centrality measures in urban traffic management by evaluating Singapore's experience, illustrating how data-driven solutions can dramatically improve the efficiency and sustainability of metropolitan transportation networks.

Case Study: Network Resilience in Tokyo During the 2011 Earthquake and Tsunami

1. Event and Disruption:

catastrophe and Interruption: This disastrous disaster seriously disrupted Tokyo's transportation network, affecting vital train and road services.

Immediate Challenges: The disaster severely damaged infrastructure, forcing the shutdown of critical highways and transit lines.

2. Application of Centrality Analysis:

Immediate Response: Tokyo used centrality criteria to swiftly analyze the disaster's impact on its transportation network. This entailed assessing which routes and nodes were most important for the city's movement and which were most affected.

Alternative Route Identification: Authorities immediately identified alternative routes that avoided damaged areas by examining the significance of key network components, ensuring crucial connectivity across the city.

3. Mitigation Strategies:

Rerouting Traffic: To avoid generating new bottlenecks, traffic was diverted to less damaged highways and bridges using centrality analysis.

Enhancing Public Transportation: Due to their centrality, certain train lines were prioritized for immediate repair, while bus services were boosted and rerouted to replace areas where rail service was affected.

4. ***Long-Term Resilience Building:***

Infrastructure enhancing: Following the accident, Tokyo concentrated on improving its infrastructure, prioritizing renovations in the most vital portions of the network using centrality measures.

Emergency Planning: The city improved its emergency planning strategies by including centrality analysis into its preparation for future potential disruptions.

This case study of Tokyo's earthquake and tsunami in 2011 clearly highlights the critical role centrality measures may play in sustaining network resilience and functionality during large disruptions, providing valuable lessons for urban planners and emergency responders globally.

Case Study: Public Transportation Optimization in Copenhagen

Background:

Copenhagen is well-known for its dedication to environmentally friendly urban development and high-quality public transit. To accommodate its rising population and satisfy its environmental aims, the city has been aggressively attempting to improve its public transportation infrastructure.

Centrality Analysis Implementation:

Identifying Central Nodes: The city identified the most critical nodes in their public transportation network using centrality measures such as Betweenness and Closeness centrality. This study looked at commuter patterns, significant destinations (such as workplaces, educational institutions, and shopping centers), and connection between various forms of transportation.

Route Optimization: Based on the findings of the centrality analysis, Copenhagen redesigned its bus and train lines to serve these major nodes more efficiently. This entailed boosting the frequency of service to high-demand areas and improving connectivity between lines.

Outcomes of the Optimization:

Increased Public Transportation utilization: By making the public transportation system more efficient and user-friendly, Copenhagen saw a rise in the utilization of its buses and trains. This was especially noticeable during peak hours, when commuters found public transportation to be more convenient than driving.

Reduced Travel Times: Route improvement resulted in significant time savings for daily commuters. Because of the increased frequency and enhanced communication between different lines, there was less waiting time and faster transit around the city.

Environmental Advantages: As more people choose public transit, there was a considerable decline in car usage, which contributed to less traffic congestion and decreased carbon emissions. This modification was in line with Copenhagen's goals of environmental sustainability and climate change mitigation.

6. Conclusion

This thesis has thoroughly investigated the use of centrality indices in the risk analysis of various transportation modes, emphasizing their critical significance in improving the resilience and efficacy of transportation networks. Degree, Betweenness, and Closeness centrality, as well as newer measures like Path Rank, Icentr and Ishortest , have proved useful in finding critical nodes and paths inside these networks. This identification is critical for comprehending the network's structural and operational dynamics, as well as identifying locations prone to disruption.

The practical uses of these indices have been proved through a variety of case studies, spanning from road networks to rail transit systems, underground transportation networks, maritime transportation networks, and air transport networks. These studies show how centrality indices can be used to analyze and manage hazards in a variety of transportation scenarios. The insights gained from this study are priceless for transportation planners and authorities, assisting in proactive network management to mitigate risks and improve performance.

However, the study acknowledges the limits of current centrality measurements, particularly their static character and the difficulties in incorporating real-time disturbances. These limitations highlight the need for future research into more dynamic centrality measures that can adapt to changing network conditions and disruptions in real-time.

In conclusion, the thesis emphasizes the importance of centrality indices in transportation risk analysis and network management. It lays the groundwork for future research aimed at improving the resilience and sustainability of transportation networks, which are critical to the operation of modern civilizations. Adopting these analytical approaches in transportation planning and management can result in more robust, efficient, and safe transportation systems that benefit society as a whole.

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